June 1895.] 1/- [No. 57.

PROCEEDINGS

OF THE

PHYSICAL SOCIETY

OF LONDON.

VOL. 13. PART 7.

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Returning now to the second half of I_2, the term not containing $\sec\chi$, we have

$$I_2''=\frac{A^2 . c^2 f^2}{2\eta b^2}\,\omega . \xi . \mathop{S}\limits_{n=1}^{n=\infty}\ \mathop{S}\limits_{m=0}^{m=n-1}\ \xi^{2n}\omega^{2m}\cos\{2(n-m)\delta-4n\psi+\phi\},$$

where we have rewritten the expressions (38) in terms of the abbreviated symbols defined by equations (36), (41), and (42). Carrying out the summation first with regard to m,

$$I_2''=\frac{A^2 . c^2 f^2}{2\eta b^2}\,\frac{\omega\xi}{1-2\omega^2\cos 2\delta+\omega^4}$$
$$\times\mathop{S}\limits_{n=1}^{n=\infty}\xi^{2n}\big[\cos(2n\delta-4n\psi+\phi)-\omega^{2n}\cos(4n\psi-\phi)$$
$$-\omega^2\cos(\overline{2n+2}\delta-4n\psi+\phi)+\omega^{2n+2}\cos(4n\psi-\phi-2\delta)\big].$$

The first and third terms in the square bracket may be summed together, and also the second and fourth. Bringing the two fractions thus obtained to a common denominator, $(1-2\omega^2\cos 2\delta+\omega^4)$ divides out, as in the case of I_2', and we obtain as our final value of I_2''

$$I_2''=\frac{A^2 . c^2 f^2}{2\eta . b^2}\,\frac{\omega\xi}{1-2\omega^2\xi^2\cos 4\psi+\omega^4\xi^4}$$
$$\times\frac{\left\{\begin{array}{l}\xi^2\cos(2\delta+\phi-4\psi)-\xi^4\cos\phi\\ -\omega^2\xi^4\cos(2\delta+\phi)+\omega^2\xi^6\cos(\phi+4\psi)\end{array}\right\}}{1-2\xi^2\cos(2\delta-4\psi)+\xi^4}. \quad (46)$$

Combining (44) (45) and (46) we have the total value of I_t, the intensity of the transmitted ray,

$$I_t=I_1+I_2=I_1+I_2'-I_2''$$
$$=\frac{A^2 . c^2 f^2}{4\eta b^2}\,\omega\xi\left\{\frac{\sec\chi(1-\xi^4)}{(1-2\xi^2\cos 2\delta+\xi^4)(1-\omega^2\xi^2)}\right.$$
$$\left.-\frac{\left\{\begin{array}{l}(1-\xi^4)[\cos\phi-\omega^2\xi^2\cos(\phi+4\psi)]\\ -2\xi^2\sin(2\delta-4\psi)[\sin\phi-\omega^2\xi^2\sin(\phi+4\psi)]\end{array}\right\}}{(1-2\omega^2\xi^2\cos 4\psi+\omega^4\xi^4)(1-2\xi^2\cos(2\delta-4\psi)+\xi^4)}\right\}. \quad (47)$$

But we want to compare this with the intensity of the ray transmitted when there is no intervening layer of electrolyte.

Calling the intensity of this ray I_0, we have for its value

$$I_0=A^2\int_0^\infty e^{-2p_1t}\sin^2 p_2t\,.\,dt=\frac{A^2}{4\eta}(\sec\chi-\cos\chi). \quad . \quad (48)$$

Dividing (47) by (48),

$$\frac{I_t}{I_0}=\frac{c^2f^2\,.\,\omega\xi}{b^2(\sec\chi-\cos\chi)}\left\{\frac{(1-\xi^4)\sec\chi}{(1-2\xi^2\cos 2\delta+\xi^4)(1-\omega^2\xi^2)}-\frac{\left\{\begin{array}{l}(1-\xi^4)[\cos\phi-\omega^2\xi^2\cos(\phi+4\psi)]\\-2\xi^2\sin(2\delta-4\psi)[\sin\phi-\omega^2\xi^2\sin(\phi+4\psi)]\end{array}\right\}}{(1-2\omega^2\xi^2\cos 4\psi+\omega^4\xi^4)(1-2\xi^2\cos(2\delta-4\psi)+\xi^4}\right\}. \quad (49)$$

This formula is one of considerable generality, showing the variation in the intensity of the ray transmitted through a thin absorbent plate when the incident ray is logarithmically damped. Two special cases are of interest: (1) when the plate is not conducting; (2) when the incident ray is not damped. Taking the first case,

$$\left.\begin{array}{lll}\text{if}\quad \alpha_2=0, & \psi=\psi'=0, & \delta=p_2t_2,\\ \theta=0, & \rho=1, & \omega\xi=b^2,\\ \xi=be^{-p_1t_2}, & c^2f^2=(1-b^2)^2, & \end{array}\right\} \quad . \quad . \quad (50)$$

and the expression for $\frac{I_t}{I_0}$ simplifies to

$$\frac{I_t}{I_0}=\frac{1-b^2}{1+b^2}\left[\frac{1-\xi^4+2\xi^2\sin 2\delta\cot\chi}{1-2\xi^2\cos 2\delta+\xi^4}\right]. \quad . \quad . \quad (51)$$

This case was recently worked out by Dr. Barton, and our equation (51) may be reduced to his. Dividing out by the denominator in the square bracket, we get

$$\frac{I_t}{I_0}=\frac{1-b^2}{1+b^2}\left[1-\frac{2\xi^2(\xi^2-\cos 2\delta-\cot\chi\sin 2\delta)}{1-2\xi^2\cos 2\delta+\xi^4}\right].$$

Dr. Barton's expression is*

$$\frac{I_t}{I_0}=\frac{1-b^2}{1+b^2}\left[1+2b^2e^{-\alpha t_2}\frac{(\alpha/\beta)\sin\beta t_2+\cos\beta t_2-b^2e^{-\alpha t_2}}{1-2b^2e^{-\alpha t_2}\cos\beta t_2+b^4e^{-2\alpha t_2}}\right],$$

which is equivalent to ours; his t_2 being equal to twice our t_2, and his α/β equivalent to our p_1/p_2 or $\cot\chi$.

---

* Proc. Roy. Soc. vol. liv. p. 92; 'Thesis,' p. 13.

As a second special case, suppose we are dealing with a steady simple harmonic ray instead of a damped wave-train. χ is then ninety degrees, and the other symbols take the special values

$$\left.\begin{aligned} &\chi=90^\circ \quad . \quad \sec\chi=\infty, \\ &p_1=0, \quad \therefore \ \omega=\xi=be^{\kappa d}, \\ &\rho=\sqrt{1+\left(\frac{\alpha^2}{p_2\beta_2}\right)^2}. \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (52)$$

The long second term in (49) vanishes altogether now, being divided by $(\sec\chi-\cos\chi)$, and

$$\frac{\mathrm{I}_t}{\mathrm{I}_0}=\frac{c^2f^2}{b^2}\frac{\xi^2}{1-2\xi^2\cos 2\delta+\xi^4}. \quad . \quad . \quad . \quad (53)$$

If in this case we again put $\alpha_2=0$, we have

$$\omega=\xi=b,$$

$$\delta=p_2t_2\,;$$

and so if the plate be a non-conductor,

$$\frac{\mathrm{I}_t}{\mathrm{I}_0}=\frac{c^2f^2}{1-2b^2\cos 2p_2t_2+b^4}, \quad . \quad . \quad . \quad . \quad (54)$$

which is the ordinary expression for the ray transmitted through a thin plate.

The general expression (49) is too complex to allow of an offhand statement of its form. One case was, however, worked out in full arithmetically. The values of the chief constants chosen were

$$\frac{\beta_2}{\beta_1}=70,$$

sp. resistance of electrolyte = 5100 ohm-centim.,

log. decrement of wave-train = 0·4,

wave-length = λ_1 = 9 metres.

These gave the following values for some of our symbols :—

$$\chi=86^\circ\ 21'\ 26'',$$

$$\theta=\ 8^\circ\ 44'\ 57'',$$

$$\kappa d = -\frac{d}{\lambda_1}(\cdot 673402),$$

$$p_1 t_2 = 3\cdot 3676\,\frac{d}{\lambda_1},$$

$$p_2 t_2 = 3017^{\circ}\cdot 825\,\frac{d}{\lambda_1},$$

$$b = \cdot 78705, \quad cf = \cdot 38167,$$

$$\log \xi = \bar{1}\cdot 8960008 - \frac{d}{\lambda_1}(3\cdot 21751421),$$

$$\log \omega\xi = \bar{1}\cdot 9920016 - \frac{d}{\lambda_1}(3\cdot 5099681),$$

$$\psi = 1^{\circ}\ 3'\ 26'', \quad \psi' = 28'\ 17'',$$

$$\rho = 1\cdot 0018.$$

The phase-changes at both surfaces are in this case so small that one would not expect any great shifting of the maxima in the transmission-curve, (1) (fig. 3): the maxima appear to occur rather early, but one can hardly say more. I find (49) too complex to get anything out of it by differentiation.

V. *The Intensity of the Reflected Ray.*

We will now proceed to calculate the intensity of the reflected ray. As before, let the incident wave be

$$\mathrm{A}\, e^{p(t-\sqrt{\beta_1}z)}.$$

A portion of this incident wave is at once reflected, namely

$$\mathrm{Y} = \mathrm{A}\,.\,b\,.\,e^{p(t+\sqrt{\beta_1}z)}\,e^{-i\psi}. \quad .\ .\ .\ .\ .\ . \quad (55)$$

A second portion enters the electrolyte and reaches the second surface in the form

$$\mathrm{A}\,.\,c\,.\,e^{p(t-qd)}\,e^{i(\theta/2-\psi')},$$

is there reflected, reaches the first surface again and emerges at time $2t_2$,

$$y_1 = -\mathrm{A}\,.\,bcf\,.\,e^{-2pqd}\,e^{p(t+\sqrt{\beta_1}z)}\,e^{i(\theta/2-2\psi'-\psi)}.$$

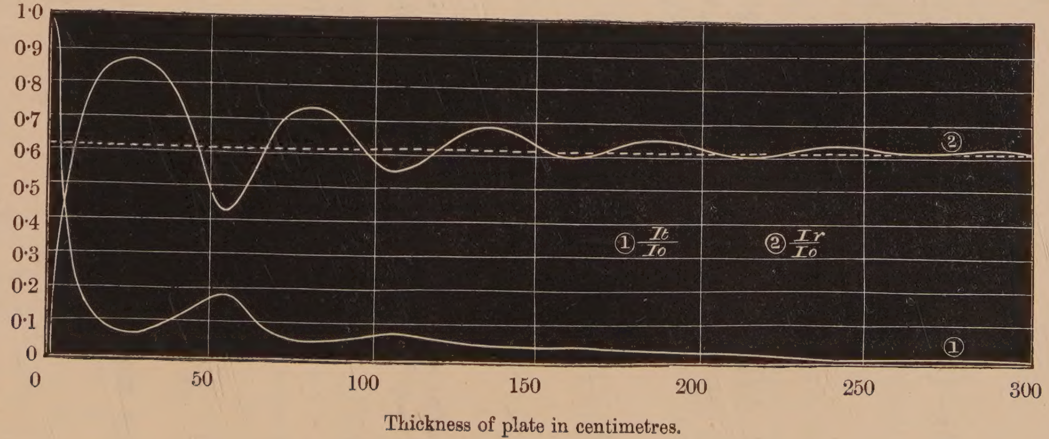
The next wave emerges after three internal reflexions at time $4t_2$,

$$y_2 = -\mathrm{A}\,.\,b^3cf\,.\,e^{-4pqd}\,e^{p(t+\sqrt{\beta_1}z)}\,e^{i(\theta/2-2\psi'-3\psi)},$$

Fig. 3.

Curves showing the variation of $\frac{I_t}{I_0}$ and $\frac{I_r}{I_0}$ with the thickness of the plate.

Ratio of dielectric constants = $\frac{\beta_2}{\beta} = 70$. Sp. resistance of plate 5100 ohm-centim.



and so on; the expression for the wave emergent at time $2nt_2$ being

$$y_n = -\mathrm{A} \,.\, b^{2n-1}\, cf \,.\, e^{-2npqd}\, e^{p(t+\sqrt{\beta_1}z)}\, e^{i(\theta/2-2\psi)}\, e^{i(2n-1)\psi}. \quad . \quad (56)$$

Now
$$pq = \eta\sqrt{\beta_2\rho}\left[i\sin\left(\frac{\theta}{2}+\chi\right) - \cos\left(\frac{\theta}{2}+\chi\right)\right].$$

Inserting this value in (56), and using our previous abbreviations,

$$\kappa = \eta\sqrt{\beta_2\rho}\cos\left(\frac{\theta}{2}+\chi\right), \quad \alpha = \left(\frac{\theta}{2} - 2\psi' + \psi\right), \quad . \quad (33)$$

$$\delta = p_2 t_2 + \psi, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (36)$$

and retaining only the sine terms, we have

$$y_n = -\mathrm{A}\, cf\, b^{2n-1}\, e^{2n\kappa d}\, e^{-p_1 t} \sin(p_2 t + \alpha - 2n\,\delta). \quad . \quad (57)$$

It is to be noted that the first reflected wave, Y, does not fall into this series so the general expression for the reflected intensity will be

$$\mathrm{I}_r = \overset{n=\infty}{\underset{n=1}{\mathrm{S}}} \int_{2nt_2}^{\infty} y_n^2\, dt + 2 \overset{m=n-1}{\underset{m=1}{\mathrm{S}}} \; \overset{n=\infty}{\underset{n=2}{\mathrm{S}}} \int_{2nt_2}^{\infty} y_m y_n\, dt$$
$$+ \int_0^{\infty} \mathrm{Y}^2\, dt + 2 \overset{n=\infty}{\underset{n=1}{\mathrm{S}}} \int_{2nt_2}^{\infty} \mathrm{Y}\, y_n\, dt. \quad . \quad . \quad (58)$$

Multiplying out $y_m y_n$ we get

$$y_m y_n = \tfrac{1}{2}\mathrm{A}^2 c^2 f^2\, b^{2(m+n-1)}\, e^{2(m+n)\kappa d}\, e^{-2p_1 t}$$
$$\times \{\cos 2(m-n)\delta - \cos[2(p_2 t + \alpha) - 2(m+n)\delta]\}; \quad (59)$$

and integrating from $t = 2nt_2$ to $t = \infty$,

$$\int_{2nt_2}^{\infty} y_m y_n\, dt = \frac{1}{4\eta}\mathrm{A}^2 c^2 f^2\, b^{2(m+n-1)}\, e^{2(m+n)\kappa d}$$
$$\times e^{-4np_1 t_2}\left\{\frac{\cos 2(m-n)\delta}{\cos\chi} - \cos[2(n-m)p_2 t_2 + 2\alpha + \chi] - 2(n+m)\psi\right\} \quad ($$

Putting $m = n$,

$$\int_{2nt_2}^{\infty} y_n^2\, dt = \frac{1}{4\eta}\mathrm{A}^2 c^2 f^2\, b^{4n-2}\, e^{4n\kappa d}\, e^{-4np_1 t_2}$$
$$\times \{\sec\chi - \cos(2\alpha + \chi - 4n\psi)\}. \quad . \quad . \quad (61)$$

The summation of (61) from $n = 1$ to $n = \infty$ is performed in exactly the same way as that of the analogous expression for I_t in equations (39) to (44). The result is

$$\underset{n=0}{\overset{n=\infty}{S}} \int_{2nt_2}^{\infty} y_n^2 dt = \frac{A^2c^2f^2}{4\eta b^2}\omega^2\xi^2$$

$$\times \left\{ \frac{\sec\chi}{1-\omega^2\xi^2} - \frac{\cos(\phi-2\psi)-\omega^2\xi^2\cos(\phi+2\psi)}{1-2\omega^2\xi^2\cos 4\psi+\omega^4\xi^4} \right\} \quad . \quad (62)$$

The summation of the product integrals in (60) is also analogous to that of the product integrals in I_t given in equations (44) to (46). Taking the integral in two parts as before, we have for the sum of the terms containing $\sec\chi$:—

$$\frac{A^2c^2f^2}{2\eta b^2}\omega^2\xi^2\sec\chi\frac{\xi^2(\cos 2\delta-\xi^2)}{(1-\omega^2\xi^2)(1-2\xi^2\cos 2\delta+\xi^4)}; \quad . \quad (63)$$

and for the sum of the second terms, those not containing $\sec\chi$:—

$$\left.\begin{array}{l} \dfrac{A^2c^2f^2}{2\eta b^2}\dfrac{\omega^2\xi^2}{1-2\omega^2\xi^2\cos 4\psi+\omega^4\xi^4} \\ \times\dfrac{\xi^2\cos(2\delta-6\psi+\phi)-\xi^4\cos(\phi-2\psi)-\omega^2\xi^4\cos(2\delta-2\psi+\phi)+\omega^2\xi^6\cos(2\psi+\phi)}{1-2\xi^2\cos 2\delta+\xi^4} \end{array}\right\} (64)$$

Adding together (62) (63) and (64), bringing the second half of (62) and (64) to a common denominator and simplifying, we get the expression for that portion of I_r which is a function of the uniform series of y's only. Calling it I'_r, and dividing out by I_0 the "intensity" of the incident ray, we get :—

$$\frac{I'_r}{I_0} = \frac{c^2f^2 . \omega^2\xi^2}{b^2(\sec\chi-\cos\chi)}\left\{\frac{(1-\xi^4)\sec\chi}{(1-2\xi^2\cos 2\delta+\xi^4)(1-\omega^2\xi^2)}\right.$$

$$-\frac{(1-\xi^4)\{\cos(\phi-2\psi)-\omega^2\xi^2\cos(\phi+2\psi)\}}{(1-2\omega^2\xi^2\cos 4\psi+\omega^4\xi^4)}$$

$$\left.\frac{-2\xi^2\sin(2\delta-4\psi)\{\sin(\phi-2\psi)-\omega^2\xi^2\sin(\phi+2\psi)\}}{(1-2\xi^2\cos(2\delta-4\psi)+\xi^4)}\right\},(65)$$

an expression very similar to that for I_t/I_0 given in equation (49). Reverting now to the terms containing Y in (58), let us evaluate

$$2\sum_{n=1}^{n=\infty}\int_{2nt_2}^{\infty} \mathrm{Y}\,.\,y_n\,dt.$$

Rationalizing (55) we have

$$\mathrm{Y}=\mathrm{A}b\,.\,e^{-p_1t}\sin(p_2t-\psi). \quad . \quad . \quad . \quad . \quad (66)$$

The expression for y_n is given in (56) : multiplying up

$$\mathrm{Y}y_n=-\tfrac{1}{2}\mathrm{A}^2cf\omega^2{}_n e^{-2p_1t}[\cos(2n\delta-\alpha-\psi)-\cos(2p_2t+\alpha-\psi-2n\delta)].$$

Integrating from $2nt_2$ to infinity, and writing for brevity

$$\Delta=\alpha-\psi+\chi, \quad . \quad . \quad . \quad . \quad . \quad . \quad (67)$$

we get

$$\int_{2nt_2}^{\infty}\mathrm{Y}y_n\,dt=-\mathrm{A}^2\,.\,\frac{cf}{4\eta}\,.\,\xi^{2n}$$
$$\times[\sec\chi\,.\cos(2n\delta-\alpha-\psi)-\cos\{2n(p_2t_2-\psi)+\Delta\}].$$

Summing this from $n=1$ to $n=\infty$, and multiplying by two,

$$2\sum_{n=1}^{n=\infty}\int_{2nt_3}^{\infty}\mathrm{Y}\,.\,y_n\,dt=-\mathrm{A}^2\frac{cf}{2\eta}\,.\,\xi^2$$
$$\times\left[\frac{\sec\chi\{\cos(2\delta-\alpha-\psi)-\xi^2\cos(\psi+\alpha)\}}{1-2\xi^2\cos 2\delta+\xi^4}\right.$$
$$\left.-\frac{\cos(\Delta+2\delta-4\psi)-\xi^2\cos\Delta}{1-2\xi^2\cos(2\delta-4\psi)+\xi^4}\right]. \quad . \quad (68)$$

Calling this I''_r, and dividing through by I_0 we get

$$\frac{\mathrm{I}''_r}{\mathrm{I}_0}=\frac{-2cf\,.\,\xi^2}{\sec\chi-\cos\chi}\left\{\frac{\sec\chi\{\cos(\psi+\alpha-2\delta)-\xi^2\cos(\psi+\alpha)\}}{1-2\xi^2\cos 2\delta+\xi^4}\right.$$
$$\left.-\frac{\cos(\Delta+2\delta-4\psi)-\xi^2\cos\Delta}{1-2\xi^2\cos(2\delta-4\psi)+\xi^4}\right\}. \quad . \quad (69)$$

The remaining portion of I_r is

$$\mathrm{I}'''_r=\int_0^{\infty}\mathrm{Y}^2\,dt=\int_0^{\infty}\mathrm{A}^2b^2e^{-2p_1t}\sin^2(p_2t-\psi)dt$$
$$=\frac{\mathrm{A}^2b^2}{4\eta}\{\sec\chi-\cos(\chi-2\psi)\}.$$

Dividing out by I_0,

$$\frac{I_r'''}{I_0} = \frac{b^2[\sec\chi - \cos(\chi - 2\psi)]}{\sec\chi - \cos\chi} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (70)$$

This last expression is independent of the thickness of the slab of electrolyte.

The total expression for $\frac{I_r}{I_0}$ is $(65) + (69) + (70)$, or rewriting these equations,

$$\begin{aligned} \frac{I_r}{I_0} = {} & \frac{c^2 f^2 . \omega^2 \xi^2}{b^2(\sec\chi - \cos\chi)} \left\{ \frac{(1-\xi^4)\sec\chi}{(1 - 2\xi^2\cos 2\delta + \xi^4)(1 - \omega^2\xi^2)} \right. \\ & \left. - \frac{\left\{ \begin{array}{l} (1-\xi^4)[\cos(\phi - 2\psi) - \omega^2\xi^2\cos(\phi + 2\psi)] \\ -2\xi^2\sin(2\delta - 4\psi)[\sin(\phi - 2\psi) - \omega^2\xi^2\sin(\phi + 2\psi)] \end{array} \right\}}{(1 - 2\omega^2\xi^2\cos 4\psi + \omega^4\xi^4)(1 - 2\xi^2\cos(2\delta - 4\psi) + \xi^4)} \right\} \\ & -2\frac{cf . \xi^2}{\sec\chi - \cos\chi} \left\{ \frac{\sec\chi[\cos(\psi + \alpha - 2\delta) - \xi^2\cos(\psi + \alpha)]}{1 - 2\xi^2\cos 2\delta + \xi^4} \right. \\ & \left. - \frac{\cos(\Delta + 2\delta - 4\psi) - \xi^2\cos\Delta}{1 - 2\xi^2\cos(2\delta - 4\psi) + \xi^4} \right\} \\ & + \frac{b^2[\sec\chi - \cos(\chi - 2\psi)]}{\sec\chi - \cos\chi} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (71) \end{aligned}$$

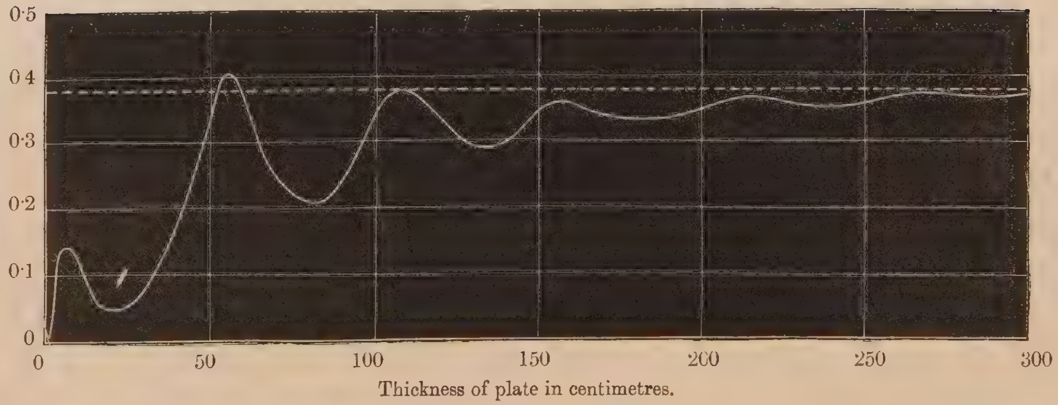
As a matter of arithmetic this expression is fairly quickly calculable if $\frac{I_t}{I_0}$ has been already worked out; a large number of the constants, &c., are the same in the two cases. The curve (2) (fig. 3) has been calculated from the same data as were given on p. 376; the two curves $\frac{I_t}{I_0}$ and $\frac{I_r}{I_0}$ may be thus compared. In the former the maxima were slightly shifted back, in the latter the minima are slightly shifted forward. The proportion of energy absorbed by the plate is

$$1 - \left(\frac{I_t + I_r}{I_0}\right).$$

The curve thus deduced from $\frac{I_t}{I_0}$ and $\frac{I_r}{I_0}$ is shown in fig. 4. As $\frac{I_t}{I_0}$ asymptotes to zero, and $\frac{I_r}{I_0}$ asymptotes to $\frac{I_r'''}{I_0}$ (70), $\left\{1 - \frac{I_t + I_r}{I_0}\right\}$ asymptotes towards the value $1 - I_r'''/I_0$; that is

Fig. 4.

Curve showing the variation in the energy absorbed by the plate $=1-\frac{I_t+I_r}{I_0}$.



to say, when the layer of electrolyte gets very thick all is absorbed but the ray reflected from the first surface.

The expression for $\frac{I_r}{I_0}$ given in (71) may be checked by putting α_2 zero. Taking the three parts (65) (69) (70) separately, the first becomes at once

$$b^2\frac{I}{I_0},$$

where $\frac{I_t}{I_0}$ is the expression given in (51) or the following equation. The second part reduces to

$$(1-b^2)\frac{2\xi^4-2\xi^2\cos 2\delta-2\xi^2\sin 2\delta\,.\cot\chi}{1-2\xi^2\cos 2\delta+\xi^4}=1-b^2-(1+b^2)\frac{I_t}{I_0}.$$

The third part gives simply b^2. Adding up we get

$$\left(\frac{I_r}{I_0}\right)_{\alpha_2=0}=b^2\frac{I_t}{I_0}+1-b^2-(1+b^2)\frac{I_t}{I_0}+b^2$$

$$=1-\left(\frac{I_t}{I_0}\right)_{\alpha_2=0}.$$

The expression for $\frac{I_r}{I_0}$ also becomes greatly simplified if we take the incident ray to be non-damped. In this case

$$\sec\chi=\infty,\quad \omega=\xi=be^{\kappa d}$$

as before, and

$$\frac{I_r}{I_0}=\frac{c^2f^2}{b^2}\frac{\xi^4}{1-2\xi^2\cos 2\delta+\xi^4}$$

$$-2cf\,.\,\xi^2\,.\,\frac{\cos(\psi+\alpha-2\delta)-\xi^2\cos(\psi+\alpha)}{1-2\xi^2\cos 2\delta+\xi^4}$$

$$+b^2.\qquad\qquad (71a)$$

If we now put $\alpha_2=0$, we have $\omega=\xi=b$, and $\delta=p_2t_2$, and so on, as given on p. 375, and the expression becomes

$$\frac{I_r}{I_0}=\frac{c^2f^2b^2}{1-2b^2\cos 2p_2t_2+b^4}$$

$$
\begin{aligned}
&- 2cfb^2 \frac{\cos 2p_2t_2 - b^2}{1-2b^2 \cos 2p_2t_2 + b^4} \\
&+ b^2 \\
&= \frac{2b^2(1-\cos 2p_2t_2)}{1-2b^2 \cos 2p_2t_2 + b^4}, \quad . \quad . \quad . \quad . \qquad (71\,b)
\end{aligned}
$$

which is the ordinary expression for 'Newton's Rings by Reflexion.'

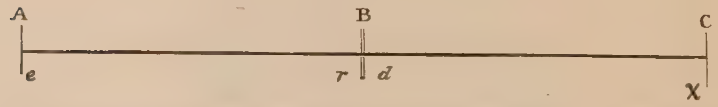
VI. *Discussion of an Experimental Case and a Correction.*

This section is independent of the rest of the paper, being devoted to the discussion of a correction necessary in the experimental case mentioned at the beginning.

It has been stated already in the introduction that the theory, sections IV. & V., does not give numerical results agreeing closely with the experimental ones. For all the values of β_2/β_1 I tried, the calculated points lay continually below the experimental determinations. Dr. Barton found exactly the same peculiarity in his analogous experiments, and suggested that it might be due to the coursing of the wave-trains backwards and forwards along the wire circuit between the oscillator and the closed end. Working from this idea he arrived at a correction formula*, which, however, is not immediately applicable to our case. The following analysis is equivalent to Dr. Barton's though differing in method.

Let ABC (fig. 5) represent our circuit, B being the electrolyte, A the oscillator, C the closed end.

Fig. 5.



Let r and d be the reflexion and transmission coefficients at the electrolyte, with regard to energy, so that

$$I_t = tI_0, \quad I_r = rI_0.$$

Let e and χ be similar coefficients for the two halves of the wire circuit—reduction coefficients covering loss of energy due either to damping by the imperfectly conducting wires or to imperfect reflexion at the ends of the circuit.

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* 'Thesis,' p. 16, section V.

If then

T_n is the intensity of the n^{th} train emerging from B towards C,

R_n is the intensity of the n^{th} train reflected from B towards A,

we have

$$\left.\begin{aligned} T_{n+1} &= ed\,R_n + r\chi\,T_n \\ R_{n+1} &= d\chi\,T_n + re\,R_n \end{aligned}\right\} \tag{1}$$

The effect on the electrometer is proportional to

$$\overset{n=\infty}{\underset{n=0}{\mathrm{S}}}\,(1+\chi)T_n,$$

so we have to sum a finite-difference series.

Let
$$\left.\begin{aligned} T_n &= Ab^n \\ R_n &= Bb^n \end{aligned}\right\} \tag{2}$$

Rewriting (1) with these values of T_n and R_n, we have

$$\left.\begin{aligned} Ab &= ed\ B + r\chi\,A, \\ Bb &= d\chi\,A + re\ B, \end{aligned}\right\}$$

whence, eliminating A and B,

$$b^2 - br\,(e+\chi) + e\chi\,(r^2-d^2) = 0. \tag{3}$$

This is an equation to determine b. Suppose b_1 and b_2 to be the roots; A_1 A_2, B_1 B_2 being the corresponding values of A and B. We have then

$$b_1+b_2=r(e+\chi), \quad b_1b_2=(r^2-d^2)\,e\chi. \tag{4}$$

$$\left.\begin{aligned} T_n &= A_1b_1^n + A_2b_2^n \\ R_n &= \frac{d\chi}{b_1-re}\,A_1b_1^n + \frac{d\chi}{b_2-re}\,A_2b_2^n \end{aligned}\right\} \tag{5}$$

From this last equation,

$$\left.\begin{aligned} T_0 &= dI_0 = A_1 + A_2, \\ R_0 &= rI_0 = d\chi\left(\frac{A_1}{b_1-re} + \frac{A_2}{b_2-re}\right), \end{aligned}\right\} \tag{6}$$

The total effect on the electrometer is consequently proportional to E where

$$E=(1+\chi)\overset{\infty}{\underset{0}{S}}T_n=(1+\chi)\left\{\frac{A_1}{1-b_1}+\frac{A_2}{1-b_2}\right\}$$

$$=(1+\chi)\left\{\frac{A_1+A_2-(b_2A_1+b_1A_2)}{1-(b_1+b_2)+b_1b_2}\right\}.$$

And from (6),

$$rI_0=d\chi\left\{\frac{b_2A_1+b_1A_2-re(A_1+A_2)}{b_1b_2-re(b_1+b_2)+r^2e^2}\right\}.$$

But by the relations (4),

$$1-(b_1+b_2)+b_1b_2=1-r(e+\chi)+e\chi\,(r^2-d^2),$$

$$b_1b_2-re(b_1+b_2)+r^2e^2=-e\chi d^2,$$

whence

$$-edrI_0=b_2A_1+b_1A_2-red\,I_0,$$

or

$$b_2A_1+b_1A_2=0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (7)$$

Thus, finally,

$$E\ =(1+\chi)\frac{dI_0}{1-r(e+\chi)+e\chi(r^2-d^2)},$$

$$E_0=(1+\chi)\frac{I_0}{1-e\chi},$$

and

$$\frac{E}{E_0}=\frac{(1-e\chi)d}{1-r(e+\chi)+e\chi(r^2-d^2)}. \quad . \quad . \quad . \quad . \quad . \quad . \qquad (8)$$

$\frac{E}{E_0}$ is an ordinate of the experimental curve. d and r are ordinates of our calculated transmission and reflexion curves. If then we determine $\frac{E}{E_0}$, d, and r for the different thicknesses of the electrolyte, we shall have by substituting their values in (8) a pair of simultaneous equations for the determination of e and χ.

This process only led to impossibilities for all values of β_1/β_2 and the log. dec. of the wave-train that were tried. Either $e\chi$ became greater than unity or negative, or $e+\chi$ became negative.

It may be noted that this correction did not work very successfully in Dr. Barton's case either. His method of applying it was, however, different: having formed an experimental estimate of the rate of decrease of the total energy of a wave-train by its passage along the imperfectly conducting wires, he assumed that there was perfect reflexion at both ends of the circuit, and thence calculated the correction to be applied to the experimental curve. The corrected curve obtained in this way lay below the calculated curve. Taking two points at random on his curves, I tried to calculate $e\chi$ and $e+\chi$ by my method. The results were again impossible.

It must be remembered that in both these cases more than one of the constants of the theoretical curve were simple guesses, γ and β_2/β_1 in my case were both only approximately known. In trying to make things square by the correction of (8), we may have been attempting the impossible and rightly got irrational results. I need not dwell at greater length on this point: possible causes of error are numerous. The non-agreement of experimental results with the equations does not of course prove the wrongness of the latter, but only the wrongness of their application to the case.

I am compelled to admit that, considering the positions of the maxima in my experimental curves are dependent on the phase-changes and on the correction we have been discussing —possibly also on other uneliminated disturbances—no great weight can be attached to the accuracy of the values of the dielectric constants for water, alcohol, &c., deduced on the assumption that such corrections were negligible *.

VII. *The Numerical Value of some of the previous Results.*

When the expressions in sections II. and III. had been obtained, the question at once arose, how far the values of b, c, f, ψ, ψ', and so on might be practically affected by possible variations in the rate of damping of the incident ray. In working with ordinary oscillators are changes likely to become important or to remain quite negligible? In working with light are differences likely to be appreciable between flashes and steady rays?

As regards oscillator wave-trains, the damping should certainly be taken into account. The second question, however, must, I think, be answered in the negative, for the simple reason that one cannot get a quick enough flash*. Even an electric spark lasts for thousands or millions of the vibrations of violet light: the rays from it would be practically steady, not damped.

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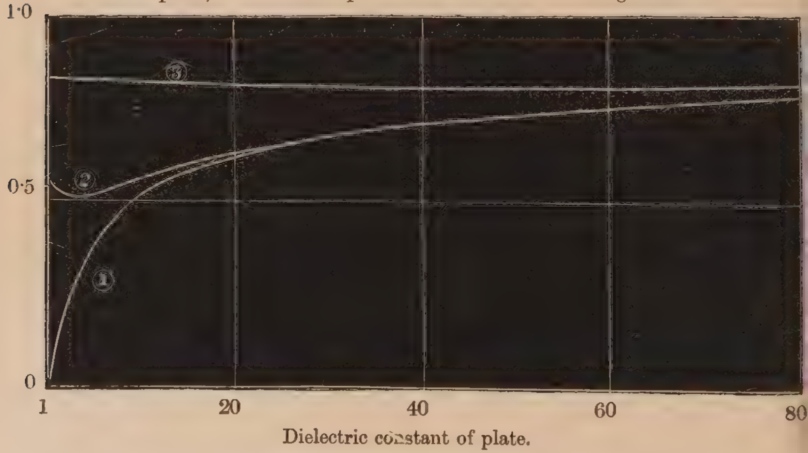
* Phil. Mag. vol. xxxvi. p. 539; Wied. *Ann.* vol. l. p. 748.

The set of curves in fig. 6 is drawn for a steady ray, wave-length one metre. It illustrates the variation in *b* (the ratio of the transmitted to the incident amplitude at the surface of an infinitely thick slab), when the conductivity and dielectric constant of the slab vary. Curve (1) is drawn for a non-conducting slab, curve (2) for a slab of conductivity $\cdot 001\times 10^{-9}$ C.G.S. units, and curve (3) for a slab of conductivity $\cdot 01\times 10^{-9}$ or ten times the last. These conductivities are all extremely low, that of a 5 p. c. copper sulphate or zinc sulphate solution being roughly $\cdot 2\times 10^{-9}$. The curves show very well how rapidly the conductivity of the reflecting plate

Fig. 6.

Curves showing the variation in "*b*" with the dielectric constant and conductivity of the plate, for an undamped wave-train. Wave-length 100 cms.



(1) For a substance of zero conductivity
(2) " " conductivity $\cdot 001\times 10^{-9}$
(3) " " " $\cdot 010\times 10^{-9}$
$\Big\}\ \beta_1=1.$

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* Unless light be itself an aggregate of damped wave-trains.

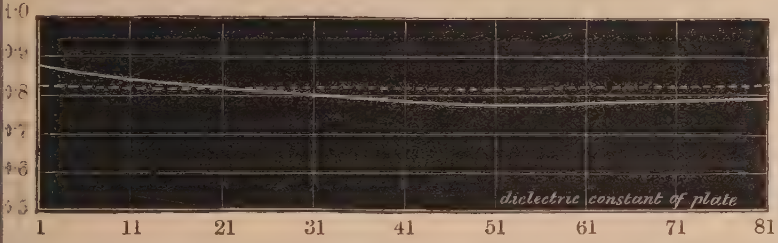
grows in importance relatively to the dielectric constant even for great wave-lengths.

But if we take the case of a charge vibrating on an isolated perfectly conducting sphere, the amplitude falls to $e^{-\frac{2\pi}{\sqrt{\ }}}$ or about $\frac{1}{38}$ of its original value in the time occupied by a complete vibration*: this gives us $\chi = 54^\circ\ 44'\ 8''$. Let us use this as an example of a damped wave-train to compare with the steady ray, retaining the same wave-length—1 metre—corresponding to a sphere about a foot in diameter.

The curve (1) for b corresponding to a non-conducting plate would be the same in both cases. The curves (3) corresponding to a plate of conductivity $\cdot 01 \times 10^{-9}$ are shown together in fig. 7. b is at first greatest for the damped wave-train, but as the dielectric constant is increased this ratio is reversed. Physically speaking, this has very little meaning: any actual method would measure the energy of the reflected ray, and it has been shown † that the energy is a function of the phase-change, the phase-change being itself a function of the rate of damping of the wave-train. Taking the phase-changes, ψ, first, I calculated them for the steady ray, for the damped ray with which we are dealing, and the conductivity $\cdot 01 \times 10^{-9}$. The results are given together in

Fig. 7.

Comparison of the values of "b" for damped and undamped wave-trains.



- - - undamped. ——— damped. $\chi = 54^\circ\ 44'\ 8''$.

Conductivity of plate $= \cdot 01 \times 10^{-9}$. Wave-length in either case $= 100$ cms. $\beta_1 = 1$.

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* J. J. Thomson, 'Recent Researches,' p. 370.

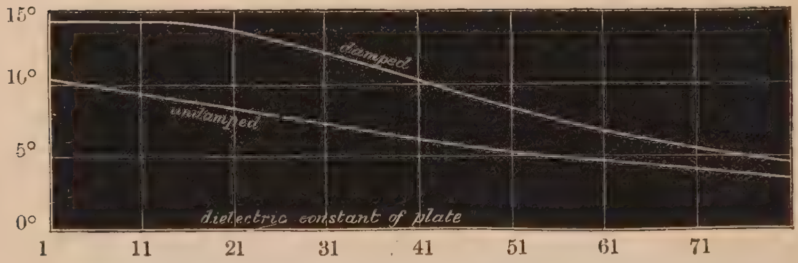
† Equation (70).

fig. 8. For the lower values of the dielectric constant, ψ for the damped wave-train is about half as large again as for the steady ray.

Utilizing these values of ψ for calculating the intensity of the reflected ray, we get the pair of curves in fig. 9. The result is striking. The energy of the reflected waves for the steady ray is from 15–20 p. c. of the incident energy greater than for the damped ray.

Fig. 8.

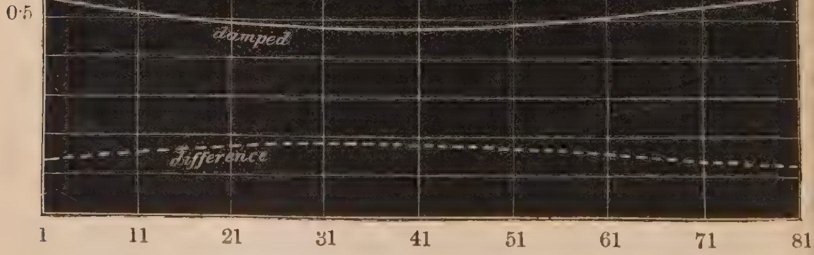
ψ for damped and undamped wave-trains.



Conductivity ·01 × 10^{-9}. $\beta_1=1$. $\lambda_1=100$ cms. $\chi=54° 44' 8''$.

Fig. 9.

Curves showing the difference between the intensities of the portions of damped and undamped wave-trains reflected from an infinitely thick plate of conductivity ·01 × 10^{-9}.



Dielectric constant of plate.

Wave-length = 100 cms. $\chi=54° 44' 8''$. $\beta_1=1$.

Experimental evidence for these results would be very interesting; but, so far as I am aware, there is none whatever. In cases where χ was taken between 80° and 90°, the alterations would be much smaller and more difficult to detect. The curves I have given only illustrate, however, a very small section of cases, even larger variations may be possible.

I am glad to have this opportunity of expressing my indebtedness to Professor Karl Pearson, to whom my best thanks are due for much advice and assistance.

TABLE *of the principal Symbols used.*

The suffix 1 refers to the dielectric; the suffix 2 to the plate. In our case, where all media are assumed to be non-magnetic,

β = dielectric constant,
$a = 4\pi \times$ conductivity.

b = ratio of the reflected to the incident amplitude at the first surface of the plate.
e = the corresponding ratio at the second surface.
c = ratio of the refracted to the incident amplitude at the first surface.
f = the corresponding ratio at the second surface.
d = thickness of slab.
λ_1 = wave-length in dielectric.
λ_2 = " " electrolyte.
$-\psi$ = the change of phase on reflexion at the first surface.
$-\psi'$ = " " refraction through the second surface.
$p = -p_1 + ip_2$ is defined by (4).
η, χ are defined by (9).
χ gives the rate of damping of the wave-train, being 90° for a steady ray.
ρ, θ are defined by (10).
r is " " (11).
κ, α are " " (33).
δ is " " (36).
ω, ξ are " " (41).
ϕ is " " (42).
Δ " " " (67).

$$I_t/I_0 = \frac{\text{intensity of transmitted ray}}{\text{intensity of incident ray}}.$$

$$I_r/I_0 = \frac{\text{intensity of reflected ray}}{\text{intensity of incident ray}}.$$

Discussion.

A letter from Dr. E. H. Barton was read emphasizing the necessity of taking into account the damping in the oscillator-train, and at the same time pointing out why, in his opinion, the corrections applied by Mr. Yule were inadequate and failed to yield intelligible results.

Prof. Rücker congratulated Mr. Yule on his work, and on the importance of the results he had obtained.

XXXIV. *Energy Movements in the Medium separating Electrified or Gravitating Particles.* *By* H. N. Allen, *University of Nebraska, Lincoln, Neb.*[*]

1. Faraday and Maxwell have shown that it is possible to look on the potential energy of electric separation as residing in the surrounding dielectric, and that each of the cells, bounded by the walls of a tube of force and two neighbouring equipotential surfaces, can be looked upon as containing a certain definite amount of energy.

This energy-distribution is not in general permanent, and can only be regarded as a step towards some simpler arrangement. Thus a positive and a negative electrified body suspended in space, and acted on only by the electrical forces between them, are never in equilibrium until they are in actual contact. The energy-distribution in the dielectric changes constantly as they approach. Poynting has shown[†] how energy is transferred from one point to another in an electromagnetic field; and we are quite accustomed to think of energy as flowing from dynamo to motor through the æther, or from primary to secondary in an alternating-current transformer.

In the following pages an attempt is made to deduce a few of the consequences of Maxwell's suggestion with regard to energy-distribution. The changes needed in the theory in order to apply it to gravitation are also indicated.

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[*] Read March 8, 1895.

[†] Phil. Trans. 1884, Pt. II. p. 343.

2. If two equal charges of positive and negative electricity $+M$ and $-M$ are separated by a distance l, the equation to the curve in which the equipotential surface V intersects the plane of the paper is

$$\frac{M}{\sqrt{y^2+(\frac{1}{2}l-x)^2}}-\frac{M}{\sqrt{y^2+(\frac{1}{2}l+x)^2}}=V, \quad . \quad (1)$$

the origin being the point midway between the particles, and the X-axis being the line joining this point to the positive particle.

3. The number of tubes of force proceeding from a charge M has sometimes been taken as M but more generally as $4\pi M$. For graphical methods the former plan seems most convenient, and in order to avoid confusion it is proposed that the resulting tubes should be called "tubes of polarization," while the smaller tubes retain the name "tubes of induction." The polarization, as defined by J. J. Thomson in his 'Recent Researches in Electricity and Magnetism,' is measured by the number of these polarization tubes which pass through a square centimetre perpendicular to their direction.

Using Maxwell's method of drawing the boundary lines between the tubes of polarization, the equation to the n^{th} line in the case mentioned above will be

$$\frac{\frac{1}{2}l-x}{\sqrt{y^2+(\frac{1}{2}l-x)^2}}+\frac{\frac{1}{2}l+x}{\sqrt{y^2+(\frac{1}{2}l+x)^2}}=2-\frac{2n}{M}, \quad . \quad (2)$$

where the straight line drawn from the positive to the negative particle is called zero, and that drawn in the opposite direction M.

4. The point of intersection of the equipotential surface V, the line of force n, and the plane of the paper lies on the curves (1) and (2). If, then, l in these equations be regarded as a variable parameter, they will together represent the curve along which the corner of an energy-cell moves, when the two particles come together. To plot this curve, the equipotential surfaces and lines of force might be drawn for a number of different distances between the particles, and corresponding points of intersection joined.

The path of the energy-cell can be obtained with less labour from measures made on a single diagram, drawn to represent the lines of force and equipotential surfaces about the two particles, when these are separated by a given distance. This is done by taking advantage of the following properties of these lines :—

1. When the distance between the particles is changed, corresponding tubes of polarization in the two diagrams are similar to one another.

2. The equipotential surfaces in the two diagrams are also similar to one another. If, however, the distance between the particles changes from l to al, the equipotential surface V changes to a similar surface, on which the potential is $\frac{V}{a}$.

If, then, the n^{th} line of force cuts the equipotential surface V at the point x, y in the first diagram, the n^{th} line will cut the equipotential surface $\frac{V}{a}$ at the point ax, ay in the second diagram.

Thus, in the case where $M=12$ and $l=10$ the coordinates of the following intersections, among others, were found by measurement on a carefully prepared diagram.

Line of force.	Equipotential surface.	x.	y.
7	1	5·15	5·95
7	4	5·35	2·28

If the diagram were enlarged in the ratio of 4 to 1, the coordinates of the point corresponding to the intersection of 7 and 4 would be $4x$ and $4y$ or 21·4 and 9·12.

The potential at this point would be $V'=\frac{V}{a}=1$.

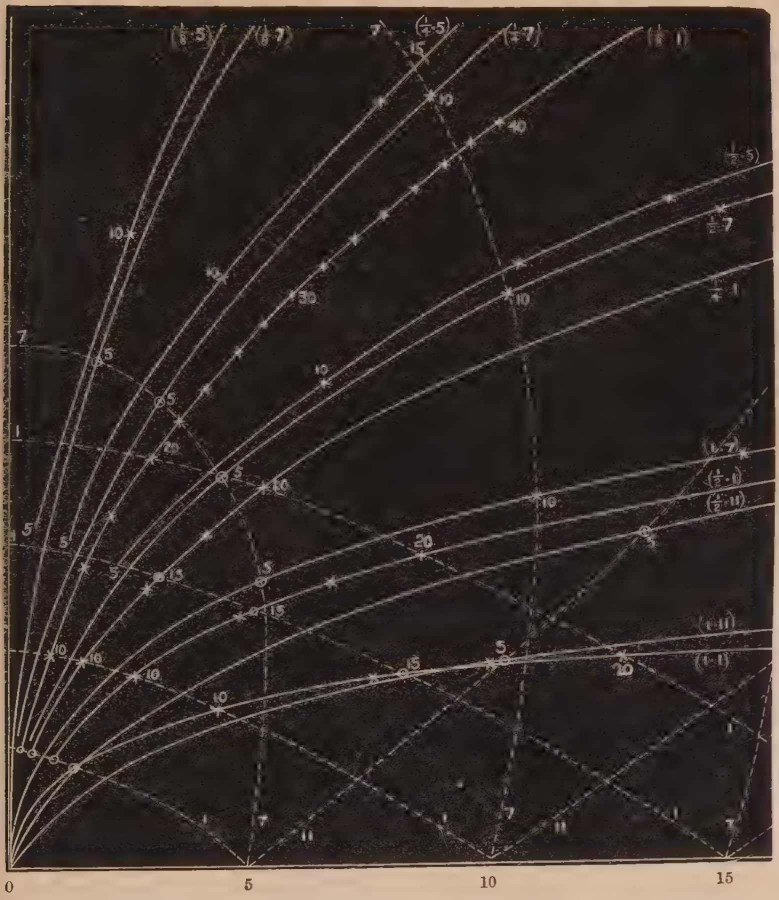
Thus we have found the coordinates of the point of intersection of the line of force 7 and the equipotential surface 1, when the distance between the particles is 40 centim.

In the same way, and by means of the same original diagram, the point of intersection can be found for a number of other distances, and the path which it follows, when the two particles approach one another, can be plotted as in fig. 1.

In this figure the first of the numbers in brackets attached to each curve gives the number of the equipotential surface, and the second that of the intersecting line of force. The numbers along the curves show the distance of the particles from the centre of gravity, when the energy-cell is at the point marked on the curve.

It will be seen that the energy-cells move in more or less

Fig. 1.

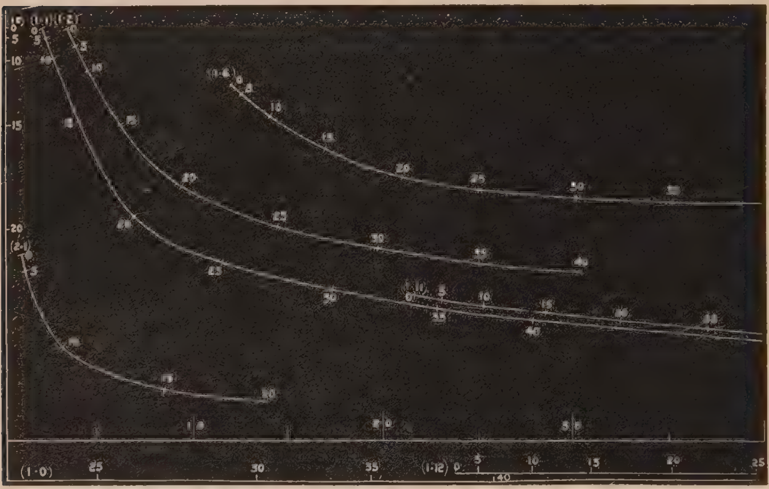


parabolic curves towards the centre of gravity of the two particles, and that during this process they are constantly approaching the two particles; so that, if these are not infinitely small, energy must be constantly passing into them from the æther.

Maxwell has shown that in all cases the number of energy-cells in the æther is twice the potential energy of the system. So that, if we suppose each cell to contain half a unit of energy, we may regard them as preserving their identity when the bodies approach, and as being absorbed by the moving bodies, so that their potential energy is converted into kinetic. A conductor moves in such a way that the absorbed energy is a maximum.

Fig. 1 also illustrates the rotation of the energy-cell as the particles approach. If on any one of the energy-paths, as for instance ($\frac{1}{2} . 7$), the direction of the line of force (indicated by a dotted line) cutting it at different points be noticed, it will be seen that this direction constantly changes as the particles approach, so that the tangent to the line of force rotates about the point of intersection. The velocity of the different energy-cells is indicated by crosses along the curves, at such points that, when the two particles come together from an infinite distance apart, the cells move from cross to cross in equal intervals of time. The same time is taken in moving from the cross marked 10 to the origin.

Fig. 2.



5. In fig. 2 the paths of the energy-cells are shown when two equal like particles repel one another. It will be noticed

that, as the particles move apart, the energy in each half of the field follows the motion of the particle in that half, and at the same time moves nearer to the central line.

It is clear that the result will be that energy will flow from the dielectric into the particle as it moves away, and being lost as potential energy will reappear as energy of motion. Even when the particles are an infinite distance apart there will still be a number of energy-cells in the æther, that is, the particles will still possess potential energy.

This is explained by supposing that every polarization tube must begin and end somewhere. In this case we may suppose that though A and B are an infinite distance apart, the negative particle C is again an infinite distance beyond A, and the negative particle D an infinite distance beyond B, so that the tubes from A end in C, and those from B in D. Then, after B and A have been repelled to a great distance from one another and all the potential energy due to their original nearness has been exhausted, there will still remain the energy due to the separation of D from B and of A from C.

6. If H_e is the intensity of the electrostatic field, or the force acting upon a particle with unit charge;
   B_e the number of tubes of induction per sq. centim., or the induction in the direction of the line of force;
   D_e the number of polarization tubes per sq. centim., or the polarization or displacement in the same direction;
   K the dielectric constant of the medium;

we have $H_e = \frac{B_e}{K}$, $D_e = \frac{KH_e}{4\pi} = \frac{B_e}{4\pi}$.

The length of an energy-cell is $\frac{1}{H_e}$ or $\frac{K}{4\pi D_e}$.

The area of section of the cell is $\frac{1}{D_e}$.

Thus the volume of the cell is $\frac{K}{4\pi D_e^2}$.

The energy-density is $\frac{2\pi D_e^2}{K} = \frac{B_e^2}{8\pi K} = \frac{KH_e^2}{8\pi}$.

The volume of an energy-cell is proportional to the square of its length measured along a line of force. Figs. 1 and 2 show how in general the portion of the line of force intercepted between two equipotential surfaces diminishes as the movement proceeds. The volume of the energy-cell constantly tends to diminish, though in some cases, as in fig. 2, the volume of some of the cells has to increase for a while to

Fig. 3.

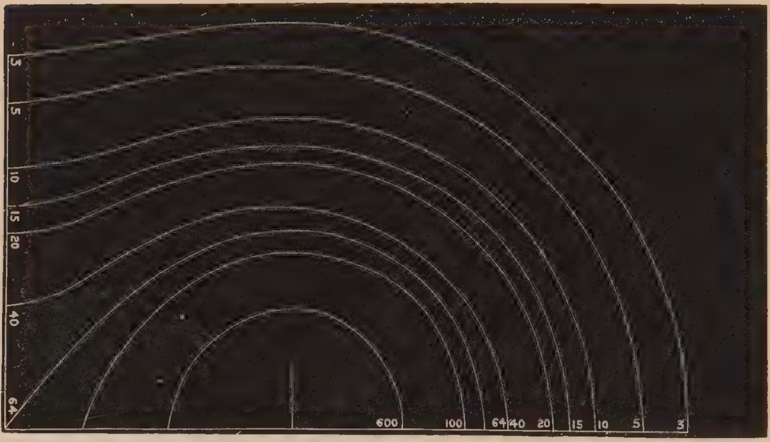
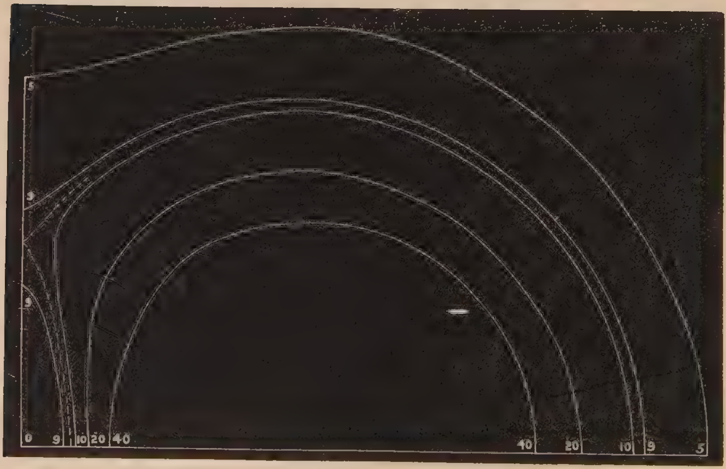


Fig. 4.



allow the rest to contract. During this contraction the length of the cell bears to its area the constant ratio $\frac{K}{4\pi}$.

It has seemed worth while to trace roughly the curves of equal energy-density in the æther about equal unlike and equal like particles. These are shown in figs 3 and 4. The numbers apply to masses of 100 with a distance 10 between them.

7. Maxwell has discussed (Scientific Papers, vol. i. pp. 570–571) the modifications which must be made in the theory to make it fit in with the observed facts of gravitation.

The energy diagram for two gravitational particles is the same as that for two like electrified particles. The polarization tubes go off to infinity, and do not, as far as we know, end on negative gravitational matter.

The properties of the cells between the polarization tubes and the equipotential surfaces are exactly opposite in the two cases of electricity and gravitation.

The electrical energy-cells are constantly tending to contract both in length and section, but so that the area of the section always bears a constant ratio to the length of the cell. The gravitational energy-cells, on the other hand, constantly tend to expand, the same ratio holding between area and length as in the former case. While each electrical cell contains according to Maxwell's theory half a unit of energy, the gravitational cells must be supposed to contain half a unit less of energy than the same volume of undisturbed æther. The gravitational cells are negative energy-cells. This expansion of the negative energy-cells implies a kind of contraction of the energy of the gravitational æther. In the case of electricity the final state is reached when the energy-density is everywhere zero, all the energy having passed into the particles; in the case of gravitation when the density is uniform over a series of concentric shells, being a maximum at an infinite distance from ponderable matter.

8. Fig. 2 shows the path of the negative energy-cells, when two equal gravitational particles come together from an infinite distance apart. It can be seen that the cells retreat from the particles as these approach one another. There will be a greater number of negative cells in the æther when the par-

ticles are in contact than when they are at a distance. This is merely another way of saying that (positive) energy has passed from the æther into the particles.

The instantaneous position of the lines of energy-flow, for a given position of the moving gravitational particles, is something like fig. 2, the motion being away from the median plane and towards the line joining the two particles. The flow of energy is on the whole in the direction opposite to the motion of the particle. The potential energy absorbed by two gravitational particles, when they fall together, comes from the space between them.

The increase in the number of negative energy-cells, when two gravitating masses approach, is equal to twice the gain in kinetic, or loss in potential energy. The proof is the same as in the case of static electricity, due regard being paid to the signs of the quantities involved.

Thus the theory may be advanced that each of these negative energy-cells contains half a unit less of energy than the same volume of æther in which no such cells exist. An increase in number of these cells corresponds to a flow of energy from the gravitational æther into matter, where it is converted into kinetic energy.

Some difficulty may arise in special cases on account of the fact that in the case of gravitation the attracting mass has a volume distribution, instead of being confined to a surface like static electricity on a conductor. Each unit mass sends out a tube of polarization, whether it is on the surface or not, and a considerable proportion of the energy-cells exist inside the surface.

In the case of a single homogeneous sphere, at a distance from all other bodies, one sixth of the negative energy-cells are within the surface.

It is clear, then, that in general the energy is not removed from the æther at the instant the negative cell passes out through the surface, but perhaps rather at the instant that it comes out from the molecule. In other words, energy may be regarded as flowing in through the surface, but is only absorbed and converted into kinetic energy when it passes into a molecule.

9. Maxwell has calculated the enormous pressure along the

lines of force and the equal tension at right angles to them required, according to his theory, to account for the attraction of the earth by the sun. A minimum value for the amount of energy in one cubic centimetre of gravitational æther can be calculated by finding the negative energy-density on the surface of the sun considered as a body of uniform density.

The volume of the energy-cells at any point is $\frac{4\pi}{H_e^2}$.

The intensity of the gravitational field on the surface of the sun is $H_e=\frac{M}{r^2}$, where M is the mass and r the radius of this body. If ρ is the density of the sun the volume of an energy-cell is

$$\frac{4\pi r^4}{M^2}=\frac{4\pi r^4}{(\frac{4}{3}\pi\rho r^3)^2}=\frac{9}{4\pi\rho^2 r^2}.$$

The amount of energy lacking in each of these cells is half a unit. Hence one cubic centimetre of æther lacks $\frac{2}{9}\pi\rho^2 r^2$ units. These are static units of energy. One dyne equals $1{\cdot}544\times 10^7$ static units of force, therefore one erg equals $1{\cdot}544\times 10^7$ static units of energy. Take r as $6370000\times 100\times 110$ centim. and ρ as $1{\cdot}39$. Then the density of negative energy at the surface of the sun will be

$$\frac{2}{9}\,\frac{\pi\times 1{\cdot}39^2\times 637^2\times 110^2\times 10^{12}}{1{\cdot}544\times 10^7}\text{ ergs per c.c.}$$

$$=4{\cdot}289\times 10^{14}\text{ ergs per c.c.}$$

$$=42{\cdot}89\times 10^{6}\text{ joules per c.c.}$$

$$=16\text{ horse-power hours per c.c.}$$

This would seem to mean that, at a distance from all gravitating matter, a cubic centimetre of æther contains at least this amount of energy.

10. That the theory may require an even greater energy-density than this is seen as follows. Suppose the energy of the gravitational æther to be due to vortex filaments or tubes of directed energy interlacing in every direction. Take at any point three axes at right angles to one another. The irregular distribution can be replaced in imagination by six equal sets of filaments parallel to the positive and negative directions of these three axes.

In space at a great distance from gravitational matter each of these sets of filaments will contain the same amount of energy. Let $6a$ be the amount of energy in one cubic centimetre of free æther. Let the positive direction of the X-axis at a point correspond with the positive direction of the gravitational line of force at that point. Let b be the volume of the gravitational energy-cell. Then five sets of filaments in the cell may contain $5ab$ units of energy, and the sixth, which has the direction of the X-axis, $ab-1$ units. Altogether the cell will contain $6ab-1$ units, while the same volume at a distance from gravitational matter will contain $6ab$ units of energy.

This would require the existence of 96 or say 100 horse-power hours in every cubic centimetre.

11. It is an interesting question whether this theory of energy-distribution in gravitational æther is or is not the simplest and most probable. That other distributions are conceivable appears evident. For example, the æther along the straight line joining the centres of two bodies might be regarded as a stretched elastic cord, the laws of contraction being of course quite different from those which hold for ordinary elastic bodies.

Another explanation which seems possible in the case of two gravitational particles is as follows. In the energy diagram for two equal electrified particles with opposite sign, suppose each infinitesimal tube of polarization to be divided into two equal tubes with opposite directions. If they are regarded as vortex filaments, each vortex will work in with its neighbours rotating the other way, like one friction-pulley on another.

Then it is clear that there is no way in which we can regard the two particles as opposed in sign, and yet if each of the two sets of vortex filaments acts as electrostatic tubes always act, the particles will be attracted together.

The energy-flow in this case will be as indicated in fig. 1, and the distribution of energy-density as in fig. 3. It has not been found possible to map the energy field in this manner for three or more particles. Indeed, the difficulties are such that it seems improbable that this method can be applied.

12. In conclusion it may be well to notice again the

assumption on which this paper is based: that the energy-cells preserve their identity, and carry the same energy with them throughout their path. It is not clear that this is necessarily true. It may be that energy passes from the potential into the kinetic form in the æther itself, and not merely on the surface of the molecules. Kinetic energy may consist of the motion of the whole system of energy-cells. This would lead us very near the theory which regards the molecule as being nothing but the mathematical centre from which forces proceed, or perhaps, from another point of view, as having infinite extension.

XXXV. *On a Simple Form of Harmonic Analyser.* *By* GEORGE UDNY YULE, *Demonstrator in Applied Mathematics, University College, London*[*].

> "The subject of the decomposition of an arbitrary function into the sum of functions of special types has many fascinations. No student of mathematical physics, if he possess any soul at all, can fail to recognize the poetry that pervades this branch of mathematics."—OLIVER HEAVISIDE.

§ 1. ABOUT a year ago several instruments for determining the coefficients of a Fourier Series expressing the equation to a given curve were described before this Society by Professor Henrici[†]. One of them, Professor Henrici's shifting-table analyser, used a planimeter as the integrator; an arrangement that seemed to me very noteworthy from the point of view of simplicity and cheapness. The analyser I am going to describe also uses a planimeter: consequently it can also only give the value of one coefficient at a time.

§ 2. Let P Q R be the curve to be analysed. Let the base P R range from $x=-l$ to $x=+l$, and the equation to the curve in terms of a Fourier Series be

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[*] Read March 8, 1895.

[†] *Ante*, p. 77; also Catalogue of the Mathematical Exhibition at Munich (1892-93).

$$y = \tfrac{1}{2}A_0 + A_1 \cos\theta + A_2 \cos 2\theta + \ldots$$
$$+ B_1 \sin\theta + B_2 \sin 2\theta + \ldots,$$

where

$$\theta = \pi x / l$$

and $\tfrac{1}{2}A_0$ is the mean ordinate of the curve. The values of the other coefficients are given by

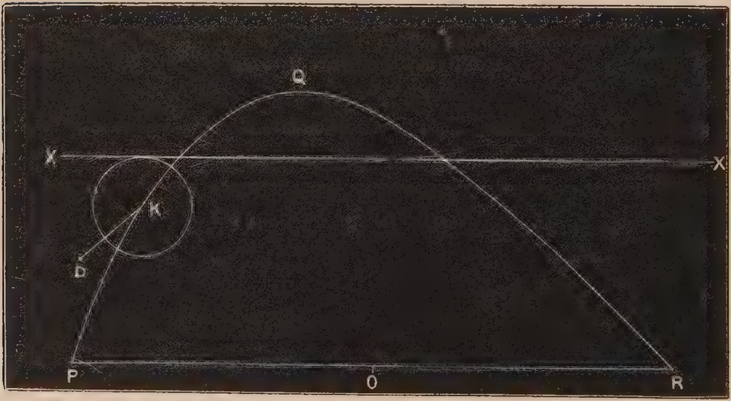
$$A_n = \frac{1}{l}\int_{-l}^{+l} y \cos n\theta \, d\theta,$$

$$B_n = \frac{1}{l}\int_{-l}^{+l} y \sin n\theta \, d\theta.$$

These are the integrals which any harmonic analyser has to evaluate.

Now suppose we have a circular disk, centre K (fig. 1),

Fig. 1.



constrained to keep in contact with a straight line X X parallel to P R, and capable of rolling along X X without slip. Further, let X X be capable of motion in the plane in a vertical but not a horizontal direction, so that every point fixed in it describes a perpendicular to P R. Then we can make the point K trace out any arbitrary curve by moving X X and rolling the disk along it.

Bring K over P, and then mark any point D on the horizontal diameter at a distance r from the centre (not necessarily inside the disk). Starting from P carry K right round P Q R and back to P again by the motion just described. Supposing the circumference of the disk to be an aliquot part of P R, say $2l/n$, let us find the area of the curve traced out by D during this operation. As the disk turns through an angle $2n\pi$ in rolling along a length $2l$ of X X, it will turn through $n\pi x/l$ in a distance x; so if x, y be the coordinates of K at some point on its journey, the corresponding coordinates of D will be

$$x-r\,.\cos n\pi\,.\cos n\theta,$$
$$y+r\,.\cos n\pi\,.\sin n\theta,$$

where, to fix the sign, we have assumed D to lie initially to the left of K and X X to lie above the disk, as in the figure.

Hence the area traced out by D is

$$\begin{aligned}R_1=&\int y\,dx-r\,.\cos n\pi\int y\,d\,(\cos n\theta)\\&+r\,.\cos n\pi\int\sin n\theta\,dx-r^2\int\sin n\theta\,d\,(\cos n\theta).\end{aligned}$$

The last two integrals vanish on taking them round a closed curve. Nothing is added to either of the first two by continuing the integration from R back to P, as y is then zero. Therefore, calling the area of the whole curve P Q R α, we have

$$R_1=\alpha+\cos n\pi\,.\frac{rn\pi}{l}\int_{-l}^{+l}y\sin n\theta\,.\,dx.$$

Similarly, if D had been initially on the vertical instead of on the horizontal diameter, and below K, we should have had

$$R_2=\alpha+\cos n\pi\,.\frac{rn\pi}{l}\int_{-l}^{+l}y\cos n\theta\,.\,dx.$$

It will evidently be convenient to take r some multiple of $1/\pi$ units of length, say 10. We then have, rewriting the last two equations,

when $r=10/\pi$

$$R_1=\alpha+\cos n\pi\,.\,10nB_n, \quad \ldots\ldots \quad (1)$$

$$R_2=\alpha+\cos n\pi\,.\,10nA_n. \quad \ldots\ldots \quad (2)$$

Care must be taken with regard to the sign on the right-hand side if any other initial position of disk and line be assumed than that dealt with above.

§ 3. These two equations contain the whole theory of my instrument; they show how to construct a curve the area of which gives the required coefficients. The geometrical mechanism seems to me to be somewhat interesting, and to be possibly capable of generalization by the use of noncircular disks.

The area of the D-curve (in dealing with a material instrument) might be obtained in two ways. We might put a pencil through the disk at D, draw the curve, and integrate it afterwards: or we might attach the pointer of an integrator to D and let the integrating go on simultaneously with the following of the curve.

It is the latter alternative that I have adopted. The former method would have some advantages, but would be slow and would lead to mechanical difficulties.

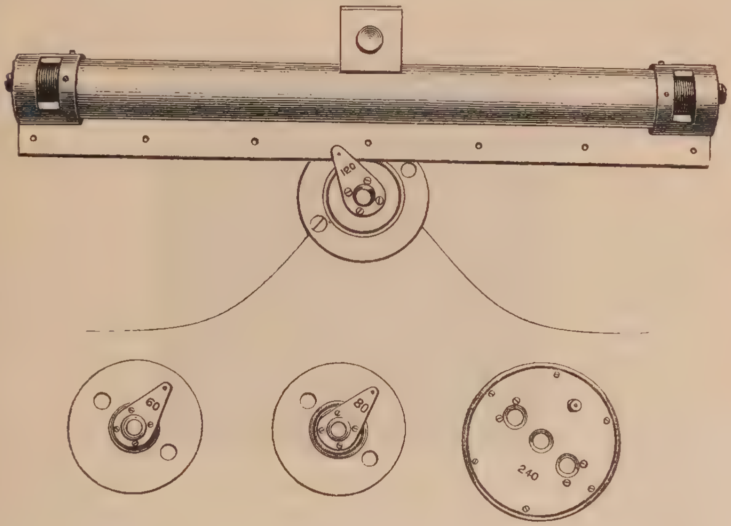
§ 4. The first analyser on this principle was made for me last summer by Mr. James Hicks. Though suffering from sundry defects (due entirely to my own design) it proved a really useful and workable analyser, but required too much care and patience in use. The construction has now been entirely revised. The present instrument, designed by Mr. Horace Darwin and made by the Cambridge Scientific Instrument Company, is the final outcome. For several suggestions I am indebted to Professor Karl Pearson.

The ruler X X of fig. 1 is a rolling parallel ruler with a rack cut along its front edge (fig. 2). The weight of the rack is counterbalanced by a block projecting from the back of the rule. Normally this block swings just clear of the paper, but it may be held down when one wants to keep the ruler still.

Corresponding to the disk of fig. 1 we have a series of toothed wheels; the number of teeth in the successive sizes being 240, 120, 80, and so on. Four of these disks have actually been made; they would probably be workable up to the sixth. The analyser is intended to work to a base-length of 30 centim.; the rack being cut 8 teeth to a centimetre. The ruler itself is made longer for the sake of stability.

The largest disk is the simplest. It is a flat disk of brass with teeth cut round the edge. Three windows are cut through it. One, in the centre, is glazed, and the centre of the glass is marked (on the side next the paper) with a black dot, the "tracing dot." The two remaining windows are provided with reference marks that give with the centre dot a base-line for setting the disk in any desired position. A small conical hole is made in the top of the disk, on a radius perpendicular to its base-line at a distance $10/\pi$ centimetres from the centre. This hole serves to receive the tracing-point of an ordinary Amsler planimeter, which performs the integration. The smaller disks are built up in three layers—a flat bottom plate, the toothed wheel, and a projecting crank, in the top of which is the planimeter hole. This hole is outside the circumference of the toothed wheel, so a crank or some such device is necessary. The crank swings clear above the rack when the wheel is in gear. The windows are arranged as in the first disk.

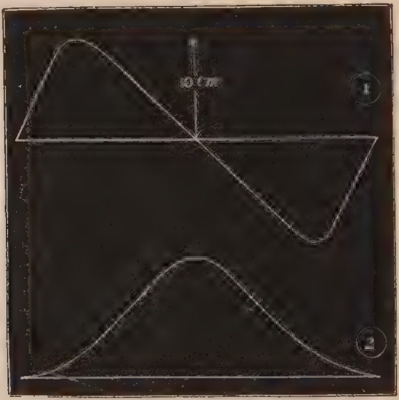
Fig. 2.



§ 5. The ordinary pattern Amsler planimeter with an arm about 16 centim. long does very well, but the tracer must be made vertically adjustable. The alteration is easily made without risk of damage. The planimeter remains of course available for its ordinary purposes, and for the determination of the absolute term.

The travel of the ruler is limited in two ways: first by the "reach" of the planimeter, secondly by the risk of running the reading wheel into the rack. Curves to be analysed must consequently be drawn to a moderate scale, but the permissible magnitude varies with the type of curve. If the type be anything like (1) of fig. 3 (sine type), 10 centim. amplitudes can be taken in comfortably; but if the type be that of (2) in the same figure, a considerably smaller scale must be used. In cases of physical curves (*e.g.* E.M.F. curves of alternators, conduction-of-heat curves) one is generally free to choose the limits of the period so as to bring the curve to the desired type.

Fig. 3.



The accuracy of the instrument will be measured by the accuracy of the planimeter. This cannot be fairly stated in percentages, as an error of unity in the vernier reading is never difficult, and may be anything per cent. in a small total. I strongly recommend drawing curves on cardboard; it is much more favourable to the planimeter than drawing-paper. The following tests may be taken as typical of the results that are obtained with care: the curves were drawn on card:—

(1) Actual curve,

$$3{\cdot}13 + 4{\cdot}60 \cos\theta + 1{\cdot}82 \cos 2\theta + {\cdot}39 \cos 3\theta + {\cdot}045 \cos 4\theta.$$

Analyser,

$$3{\cdot}14 + 4{\cdot}58 \cos\theta + 1{\cdot}84 \cos 2\theta + {\cdot}39 \cos 3\theta + {\cdot}042 \cos 4\theta.$$

(2) Actual curve (sloping straight line),

$$6{\cdot}37 \sin\theta - 3{\cdot}18 \sin 2\theta + 2{\cdot}12 \sin 3\theta - 1{\cdot}59 \sin 4\theta.$$

Analyser,

$$6{\cdot}39 \sin\theta - 3{\cdot}20 \sin 2\theta + 2{\cdot}11 \sin 3\theta - 1{\cdot}58 \sin 4\theta.$$

The units are centimetres.

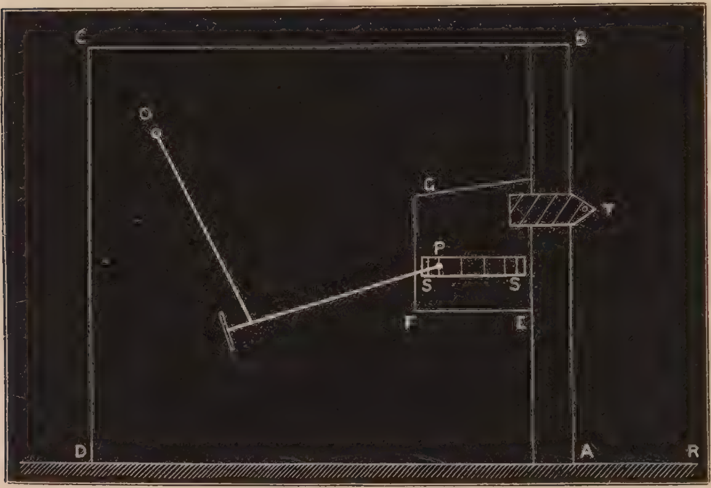
§ 6. So much for the accuracy and range of the instrument. To get any desired coefficient, the ruler is set with its edge parallel to the curve-base and with the proper disk in gear. Ruler and disk are then adjusted till the tracing-dot stands over the point P (fig. 1), and the base-line of the disk is either vertical or horizontal according as a sine or cosine term is wanted. The planimeter-point is finally dropped into the hole provided for it, and the tracing-dot carried completely round the curve. The resulting planimeter reading will be the area of the curve plus or minus $10\,n$ times the desired coefficient.

Both hands must be used in guiding the disk, as rack and disk have to be held together while the latter is turned. The operator forms, in fact, an essential link of our mechanism which without him is unconstrained. It is this liberal use of the operator that enables me to dispense with slides, carriages, and other expensive things, and thus gain in simplicity.

§ 7. This analyser arose from a simple form of step-by-step integrator or "adder" which may be worth a brief description. The instrument is shown diagrammatically in fig. 4: it was made from materials at hand and I describe it as made. A B C D is a square sheet of card with a foot-rule glued along one edge A B. A set square F E G can be slid up and down along this rule: it is provided with a tracer T and some scales of sines S S. The whole sheet of card is guided parallel to the axis of x by a T-square D A R clamped to the drawing-board: if the card is pulled forward by the corner A the friction keeps it set against the square. O P is a planimeter with its pole fixed to the card and its pointer P resting on one of the scales of sines, which must stand parallel to the axis of x.

Mark off along the base of the curve to be analysed a number of equal divisions, *e. g.* 6° each, and erect the ordinates y_1, y_2, &c. at the centres of each of these elements. Suppose the pointer P of the planimeter to rest initially at the zero of the scale, and the tracer T attached to the set square to stand over the origin of the curve. Pull the card forward, carry T to the top of y_1, and then shift P to 6° on the scale. Pull the card forward again, carry T to the top of y_2, and then shift P to 12°. Continue this procedure right on to the end of the curve.

Fig. 4.



P will then have come back to its starting-point on the scale, after describing on the card a certain curve or stepped polygon. The area of the polygon is

$$\begin{aligned} &y_1 (\sin 6^\circ - \sin 0^\circ), \\ &+ y_2 (\sin 12^\circ - \sin 6^\circ). \\ &+ y_3 (\sin 18^\circ - \sin 12^\circ), \\ &+ \quad . \quad . \quad . \quad . \quad . \quad . \quad . \\ &+ y_4 (\sin 360^\circ - \sin 354^\circ), \end{aligned}$$

a quantity which (as the elements are small) approximates to

$$\int_0^{2\pi} y \, d(\sin\theta).$$

Thus the planimeter-reading after this procedure gives us the first cosine coefficient. I need not enter at length into the mode of getting the others. For the second coefficient one would have to shift the tracer P 12° at a time instead of 6°, and so on.

In my case I actually used steps of 6°, as above; and there were three scales on E F G going by steps of 6°, 12°, and 18° respectively for the first three terms, the separation of the scales helping to avoid confusion. The results were good: for example, in one test the actual coefficients were 4·82, 1·09, ·009: the instrument gave 4·86, 1·08, ·01. The chief objection to such a non-automatic integrator is of course that one is liable to forget to shift the planimeter pointer at some stage of the proceedings. The chief advantage is that your curve need not be drawn to any particular base-length. Whatever the base, it is only necessary to divide it into the proper number of equal parts, and erect the ordinates at the centres of these elements.

Suppose we wished to make the arrangement automatic. We might substitute for the harmonic motion of P along S S a circular motion round the centre of S S. This merely amounts to giving P another harmonic motion (perpendicular to S S), a proceeding which adds nothing to the planimeter-reading if the integration be continued completely round the curve.

But this is not an easy motion to obtain mechanically. The difficulty is obviated at once if we remove the card A B C D altogether and fix the pole of the planimeter in the drawing-board. If we give P the same circular motion as before we have the "disk" analyser, which I described in the first part of this paper. The area of the curve analysed is added to the integral given by the planimeter, but that is all.

Evidently in the instrument of fig. 4 we have only taken a special case in making the scales scales of sines. We might have used scales graduated proportionally to x^n and got moments. Any other integral could be obtained approximately if a proper scale could be drawn.

DISCUSSION.

Prof. HENRICI said he had at one time considered the question of constructing an analyser which should employ a planimeter as the integrator, and he was particularly pleased with the instruments exhibited. Since the area required was the difference between the area of the original curve which is traced out by the centre of the disk (K) and the curve traced out by the point D, and since this area is really the area swept out by the straight line KD, if we attach an integrating wheel to the disk with its axle parallel to KD, the required area can be directly obtained from the reading on this wheel. In addition, if a second integrating wheel were fixed to the disk with its axle perpendicular to KD, the coefficients of $\cos n\theta$ and $\sin n\theta$ could be both obtained by going round the curve once. The instrument devised by Mr. Yule was practically the inverse of one he (Prof. Henrici) had invented.

Dr. BURTON pointed out some incorrect signs in the proof given. These, however, do not affect the final expressions obtained.

Mr. INWARDS suggested that errors due to back-lash might be avoided by using either a double wheel or a double rack, so that by means of a spring each side of the teeth which were engaged might be in contact at the same time.

XXXVI. *On the Objective Reality of Combination Tones.* By

A. W. RÜCKER, *M.A., F.R.S., and* E. EDSER, *A.R.C.S.*[*]

THE question of the objective existence of combination-tones has been keenly disputed. At first they were regarded as produced within the ear itself. Von Helmholtz in part adopted this view, and gave a theoretical explanation of the way in which the construction of the ear might lend itself to such a result. ('Sensations of Tone,' App. XII.) He also believed that they existed objectively when the amplitudes of the vibrations of the notes which give rise to them are so great that powers higher than the first have to be considered. He supported this view by mathematical demonstrations, and stated ('Sensations of Tone,' transl. by A. Ellis, p. 157) that he had proved their objective existence

[*] Read March 22, 1895.

by making membranes and resonators to respond to combinational tones produced by the siren and harmonium. These views and statements have been adversely criticised by König, Bosanquet, and Preyer. A very lucid account of the controversy was given by the late Mr. Ellis in his translation of the *Tonempfindungen*, and the net result of the impression produced in his mind is shown by two notes on pages 156 and 157. He there states that the result of Mr. Bosanquet's and Prof. Preyer's experiments is to show that the combinational tones are produced in the ear itself, and that it is probable that the apparent reinforcement of the resonators noticed by Helmholtz arose from imperfect blocking of both ears when using them.

These statements were unqualified, and no condition was made as to the way in which the combination-tones were produced. Helmholtz, for reasons which we need not recount, regarded the siren as the best instrument for producing objective combination-tones ; and we recently determined to submit the question of their existence, which seemed to be decided against him, to another experimental test.

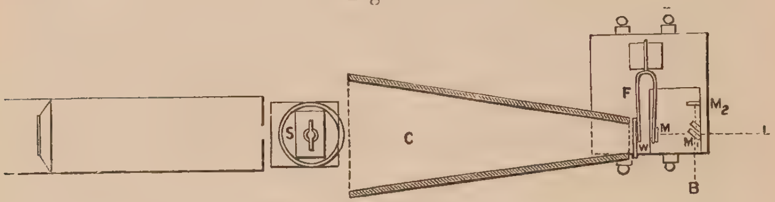
In this paper we give the result of our investigations, as far as they have at present been carried out. We do not regard them as complete, but they at all events prove that when the conditions under which we experimented are fulfilled, there can be no doubt that difference- and summation-tones are produced which are capable of disturbing resonating bodies.

The resonator employed in the first instance was a tuning-fork. It is well known that this instrument is relatively difficult to excite by resonance, and it was therefore necessary to use an extremely delicate method of detecting whether it was set in motion. For this purpose a mirror attached to one of the prongs was made one of a system by which Michelson's interference-bands were produced. A movement of the prong amounting to half a wave-length of light (say 1/80,000 of an inch) would alter the length of the path of one of the interfering rays by a wave-length. A periodic vibration of this amplitude would cause the band to disappear.

It is therefore evident that an extremely minute movement could be detected. It was at first open to question whether the apparatus would not be so sensitive to accidental disturbances as to be untrustworthy. This difficulty has, however, been entirely overcome.

A plan of the apparatus is shown in fig. 1. The shank of the tuning-fork, F, is imbedded in a mass of lead. One of the prongs carries a mirror, M. To the other is attached a square of wood, W, of larger area but of the same weight as the mirror. A Lissajous' figure (an ellipse) was formed by reflexion from the mirror and from a small square of silvered glass attached to one of König's large forks adjusted to give the C of 64 complete vibrations per second, and the pitch of F was adjusted until only one beat occurred in two minutes. The pitch was thus very accurately known. A double siren, S, was placed between a large König resonator tuned to 64 vibrations and a wooden cone or pyramid, C. The end of the cone was placed about half an inch from W, which was rather larger than the narrow end of the cone. The sensitiveness of the apparatus depended in part upon the distance between W and C. If the distance was too large, the sensitiveness was diminished. If it was too small, the instrument was unduly affected by chance puffs of air even when not periodic in character.

Fig. 1.



A source of light, L, was used to form a system of interference-bands by means of the half-silvered mirror M_1, and the two mirrors M and M_2. The two interfering rays travelled over the paths $LM_1M_2M_1B$ and LM_1MM_1B respectively. The distances MM_1 and M_1M_2 were approximately equal; and since when soda-light was used a change in the length of either of the paths of $\frac{1}{2} \times 589\,\mu\mu$ would cause a dark band to shift into the position previously occupied by the next bright band, and since, further, any movement of M altered the length of the path of the ray by twice its own magnitude, it is evident that a movement of $\frac{1}{4} \times 589\ \mu\mu$, or, say, of one hundred thousandth of an inch, could be easily detected.

The fork F and the mirrors M, M_1, and M_2 rested on a square stone, which was suspended by wires and india-rubber door-fasteners from a heavily weighted beam, which itself rested on india-rubber balls placed at a convenient height on a double pair of wooden "steps." With these precautions it was found that the bands remained tolerably steady even by day when persons were moving about the building, and when the traffic on the frost-bound road produced considerable mechanical vibrations. Even under these conditions, we have satisfied all who have seen the apparatus of the reality of the phenomena. The experiments on which we rely, however, were made on several occasions between midnight and 2 or 3 A.M. The bands were then absolutely clear and steady. They were undisturbed for many minutes at a time when the bellows were being worked and the siren was sounding loudly. It was only when a note of 64 vibrations per second was directly or indirectly produced that they vanished, and there could be no possible doubt or mistake as to whether the disturbance was or was not produced by the sound or combination of sounds under investigation.

Up to the present we have used the fork above described only. It was chosen because it was fairly stiff, and as removable metal mirrors for the production of Lissajous' figures were attached to its prongs, it was possible to replace them by the glass mirror and square of wood without altering its pitch. It would be quite possible to use properly made forks of higher pitch as resonators, and the steadiness of the bands at night is so remarkable, that we believe that if the apparatus were set up in the country, on a stone isolated from the rest of the room, the degree of sensitiveness we have attained could be far surpassed.

Tuning the Siren.

Three methods were used for determining when the siren was producing the required notes. When one of these was fairly high, the beats given by it and a standard fork were noticed, and the note could thus be kept hovering about the required pitch for a considerable time. Although with the aid of König's large forks we could apply this method to vibration-frequencies of 48 per second and upwards, it was difficult when the notes were very low to recognize the beats with sufficient certainty.

The prongs of various tuning-forks were therefore furnished with two pieces of tin-foil, which opened and closed a slit made in them twice in every complete vibration. They were also compared with a standard by the aid of a revolving cylinder, and were adjusted by weights to the required frequencies. These were so selected as to make one of the circles of holes on the siren appear stationary when viewed through the slit, if the velocity of rotation was such that the desired note or notes would be given by the same or other circles of holes. An observer watching the siren through the slit and pressing lightly on the axis with a straw, was able to adjust its speed so that the required combination of sounds was produced for several, and in some cases for many seconds at a time.

We have also projected the image of the row of holes on a screen, the cone of light passing at its narrowest part through the slit in the tinfoil screens carried by the auxiliary fork. When the upper siren was to be used, two semicircles of light mirror-glass were laid on the disk of the lower siren to which the two halves of a paper cog-wheel were fastened. The image of the cogs could thus be projected.

The third method of determining the speed of the siren depended on the use of an instrument first devised by Lord Rayleigh in 1880. A mass of air enclosed in a tube is excited by resonance, and the fact of the excitation is indicated by a light mirror set where the motion is greatest, and inclined at 45° to the direction of the air-currents. In accordance with the general law that a lamina tends to place itself perpendicular to the direction of a stream, the mirror moves when the air vibrates. In the original apparatus the amount of the movement was controlled by magnets. Since that date Professor Boys has modified the instrument by substituting a quartz-thread suspension for a silk fibre, and using the torsion of the thread instead of the directing force of the magnets. He exhibited the apparatus during a lecture delivered before the British Association in Leeds. It may be called a mirror-resonator.

Professor Boys has been good enough to make two of these instruments for us, and it was decided that one of them should respond to 161 vibrations per second. The movement of the spot of light reflected from the mirror informed us when the siren was giving this note, and this fact was utilized in one of the experiments.

Sensitiveness of the Apparatus.

The results were in general improved by laying a small strip of dry blotting-paper upon the prongs. Mere accidental disturbances died out more quickly, and the vibrations of the fork diminished more rapidly, when the notes which had produced them ceased to sound.

A forced movement could be produced in the fork by blowing directly down the collecting cone, but a small organ-pipe could be sounded, or a König's fork bowed near the opening without producing the least effect if the frequency of the note produced was not near 64 per second. In like manner many notes of two reed wind-instruments, somewhat similar in construction to the harmonium, were sounded simultaneously so that the room echoed with discordant sound, yet without producing the least effect on the steadiness of the bands.

On the other hand, when one of König's forks tuned to 64 vibrations was touched with the india-rubber covered handle of a gimlet, or was struck with a piece of gas-tubing so lightly that an observer with his ear close to the fork could not detect the fundamental note, the bands instantly disappeared.

It was therefore evident that the apparatus could respond to a vibration which was quite inaudible to a person standing by the resonating fork.

When the König fork was weighted so as to give 63·5 vibrations per second, the bands appeared and disappeared at regular intervals of two seconds, thus corresponding to the beats between the exciting and resonating forks. The bands were not disturbed when the beats exceeded 2 or 3 per second unless the exciting fork was bowed very violently. In the experiments with fainter tones, the resonating fork was only disturbed when the frequency of the exciting note agreed very exactly with its own.

As the speed of the siren was gradually increased from rest the bands always disappeared two or three times, the disturbance being no doubt caused by the upper partials as the vibration-frequency passed, in turn, the values 16, 21·3, and 32 per second. No exact measurement was made of the corresponding frequencies, as there could be no question as to whether the partials were produced or as to whether the apparatus could detect them. No such disappearance occurred when the note was higher than C, except in the case of certain spurious effects referred to below.

All the experiments were performed several times on each occasion, and on two occasions at least. The bands were sometimes produced by soda-light, and watched by an observer through a telescope.

Sometimes the electric light was used. The bands could then be projected on a screen side by side with the image of the holes or cogs. In this way information as to the note produced and as to the behaviour of the bands was simultaneously conveyed to all who watched the experiments.

Precautions.

It will not be thought remarkable that so delicate an apparatus requires careful use.

In the first place, it was necessary to be sure that the disturbance ascribed to the joint action of two notes was not produced by one or both of them when acting alone. In all cases, therefore, in which it was possible to determine the pitch of both notes separately, each note was sounded alone and carried slowly past the particular pitch at which the existence of a combination-tone was to be investigated. In general, no effect was produced; but after the apparatus had been in use for some little time, we were troubled by the fact that two notes which did not differ much from 256 and 320 vibrations respectively disturbed the bands when they were produced separately.

That these effects were spurious was evident from two facts. Firstly, the same notes had previously been sounded for a long time with the special object of determining whether they produced any effect, and had produced none. Secondly, the disturbances were evidently forced. When the fork has been disturbed by a vibration of frequency 64, it continues to oscillate for several seconds after the disturbing note has ceased. The bands alternately appear and disappear several times before the final state of rest is attained. In the case of a "spurious" effect, the bands reappear instantaneously with absolute clearness the moment the disturbing note ceases, thereby indicating a forced vibration. It was, however, a more difficult task to discover the cause than to determine the character of these disturbances. Finally, it was found that since the apparatus was first set up some cracks had opened in the wooden pyramidal collector. A tap on the side of this will disturb the bands; and when one of the cracked sides was loaded with weights the disturbance due to the 320 note was much reduced. The cracks were then cut out, the defects made good; and since that time this disturbance has entirely ceased.

The effect produced by the 256 note was due to another cause. When the pyramid was removed, the open ends of the resonance-boxes of various tuning-forks (256, 320, 384) were held near the square of wood which was attached to the fork F. It was found that the 256 fork was the only one which affected the bands, and that it only produced any result when the sounding-box was held near that part of the wood which projected beyond the end of the prong. It was therefore evident that the two ribs attached to this were not sufficient to prevent a certain flapping of the projecting part, and that this responded to a note of about 256 vibrations. The end of the collecting pyramid was then shifted a little so that the waves of sound impinged only on the more rigid part of the wooden square, and the spurious effect immediately and completely disappeared.

Of the notes which caused these troubles, the lower one was decidedly lower than the c' of 256; the other corresponded very accurately with the e' of 320. The fact that both these numbers are multiples of 64 may legitimately give rise to the doubt whether the fork F can be set in motion by disturbances which are multiples of its own frequency.

Even if it be admitted that such forced vibrations would be especially easy to produce, we must insist—(1) that we can distinguish (as above described) between forced and natural vibrations ; (2) that when the apparatus was properly arranged such notes could be produced by the siren for long intervals of time without the least effect on the bands ; (3) that two tuning-forks, of 256 and 320 vibrations respectively, have been placed within the wooden pyramid with their sounding-boxes resting on and supported by, the lower side, yet no effect was produced on the bands even when they were both sounding loudly at the same time ; (4) that one of König's large forks, with a frequency of 128, was placed in front of the collecting cone and produced no effect when sounding loudly, though the slightest tap on the 64 fork caused the bands to vanish for many seconds ; (5) that the 320 and 384 forks produced no effect when the pyramid was removed and they were held as above described close to the receiving prong of the fork ; (6) and, lastly, that in two out of the five experiments on the difference-tones, and in all those in which the interference apparatus was used to detect summation tones, the frequencies of the notes employed were neither multiples nor sub-multiples of 64.

As in all the above experiments the resonator was in position, they also sufficed to prove that tones of 64 vibrations were not manufactured in it by the primary notes when acting singly.

If the objection is raised that, although neither note disturbed the bands when sounded alone, the effect might be due to the double disturbance produced by the two sounds, we think it sufficient to answer that we have always carried the two notes above the pitch at which the difference-tone might be expected to affect the instrument, then lowered the pitch again till the notes were too flat, and repeated this operation several times. The disturbances were only produced when the pitch was nearly, or very nearly, correct As soon as a few beats per second were heard the bands became visible.

Having thus described the various tests which were applied to the apparatus, we may proceed to describe the experiments without dwelling further on the necessary precautions. It is

PHYSICAL SOCIETY OF LONDON.

ABSTRACTS OF PHYSICAL PAPERS

FROM FOREIGN SOURCES.

JUNE 1895.

GENERAL PHYSICS.

410. *Integration of the Equations of Equilibrium of Elastic Bodies.* **G. Lauricella.** (N. Cim. 4. 1. pp. 155–165, 1895.)—The following problem is discussed :—Let there be an elastic body whose boundary is a closed surface S, convex at every point, and whose tangent-plane varies continuously with variation of the point of contact, and three functions $u(m)$, $v(m)$, $w(m)$, whose values are arbitrarily given for each point m on S. It is required to find three functions U, V, W which shall satisfy the equations of equilibrium at each point n within S, and as n approaches m on S shall tend to assume respectively the values $u(m), v(m), w(m)$. Let $x_1\, y_1\, z_1$ be the point n within S, r the distance from n to m. Then a, b, c are three functions of r defined in the preceding memoirs which satisfy the equations of equilibrium for all positions of n, and also satisfy the conditions

$$\iint a\, d\mathrm{S} = 4\pi, \quad \&\mathrm{c}.$$

Assume

$$\mathrm{U}_1(n) = \frac{1}{2\pi}\iint \{u(m)a + v(m)b + w(m)c\}\, d\mathrm{S}.$$

As n approaches m let a, b, c be denoted by a', b', c'. It is shown that

$$\text{limit } \mathrm{U}_1(n)_{n=m} = u_1(m) + u(m),$$

in which

$$u_1(m) = \frac{1}{2\pi}\int \{u(m)a' + v(m)b' + w(m)c'\}\, d\mathrm{S}.$$

Two other functions $V_1(n)$ and $W_1(n)$ are treated in the same way, from them being derived $v_1(m)$ and $w_1(m)$. But the three new functions $u_1(m)$, $v_1(m)$, $w_1(m)$ can be used for $u(m)$, $v(m)$, $w(m)$. And so we find

$$U_2(n) = \frac{1}{2\pi}\iint \{u_1(m)a + v_1(m)b + w_1(m)c\}\, dS\,;$$

and

$$\text{limit } U_2(n)_{n=m} = u_2(m) + u_1(m).$$

Similarly

$$\text{limit } U_0(n)_{n=m} = u_3(m) + u_2(m),$$

and so on. Hence he forms the three series,

$$\begin{aligned} U &= \{U_1(n) - U_2(n)\} + \{U_3(n) - U_4(n)\} + \ldots \\ V &= \{V_1(n) - V_2(n)\} + \{V_3(n) - V_4(n)\} + \ldots \\ W &= \{W_1(n) - W_2(n)\} + \{W_3(n) - W_4(n)\} + \ldots \end{aligned}$$

which satisfy the equations of equilibrium at all points within S, and as n approaches m assume the forms

$$\begin{aligned} &\{u(m) - u_2(m)\} + \{u_2(m) - u_4(m)\} + \ldots \\ &\{v(m) - v_2(m)\} + \{v_2(m) - v_4(m)\} + \ldots \\ &\{w(m) - w_2(m)\} + \{w_2(m) - w_4(m)\} + \ldots \end{aligned}$$

It is then shown that both sets of series are convergent. Also the second set becomes

$$\left.\begin{aligned} u(m) - C, \\ v(m) - C, \\ w(m) - C, \end{aligned}\right\}\text{ where C is a finite quantity.}$$

And therefore the three functions $U + C$, $V + C$, $W + C$ satisfy all the conditions of the problem. S. H. B.

411. *Internal Friction of Metals* (Part I.). **M. Cantone.** (N. Cim. 4. 1. pp. 165–183, 1895.)—Experiments were conducted on wires of about 2 mm. diameter carrying a cylinder of considerable moment of inertia, arranged so that they could either be carried through cycles of torsional stress, carried out slowly, step by step, or allowed to oscillate torsionally to observe the damping. The strains were observed by a telescope and scale. A static cycle was first observed, and the wire then allowed to vibrate to rest from its position of maximum strain. Tables are given, showing the value of $L = \int M\, d\alpha$ for the cycle, where M is the torque for twist α, so that L is the work spent in the cycle, and L′ the energy lost per vibration in the dynamical experiment: air-friction was found to be negligible. Results are given for brass and iron. The most important conclusions are

that (i) repeated straining beyond the limits of elasticity produces an "accommodation" in the metal, tending to make its behaviour approximate to Hooke's law, with an elasticity slightly greater than that at first observed for very small strains; (ii) for strains small enough to obey Hooke's law sensibly, there is still some loss of energy by hysteresis, and apparently the state of "perfect elasticity" is never reached exactly; (iii) "accommodation" alters the hysteresis-loss and the damping very much. Both these quantities are diminished, especially in the case of brass; so that in one instance the wire took 52 vibrations to lose as much in amplitude as, at first, it lost in 9. The two quantities of energy are not equal, even after the wire has suffered much treatment, the damping being usually less than loss in a static cycle, except when the strains are very great: they are always, however, of the same order of magnitude, and the author concludes that static hysteresis is the chief cause of damping. Altering the period of oscillation did not alter the damping per oscillation; so there does not seem to be a time-lag. Iron differs from brass chiefly on account of its greater range of approximately perfect elasticity. R. A. L.

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412. *Reduction of Weighings to Vacuo.* **P. Schottländer.** (Zschr. phys. Chem. 16. pp. 458–462, 1895.)—Some considerations on the approximate correction of weighings made in air. The formulæ given are practically identical with those which are already well known (*cf.*, for example, Stewart and Gee, Pract. Physics, i. p. 89). T. E.

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413. *Determination of the Molecular Magnitudes of some Inorganic Substances.* **H. Biltz.** (Sitz. Akad. Wiss. Berlin, pp. 67–90, 1895.)—The molecular magnitudes of organic substances are now frequently determined by examining the properties of their solutions; the method is easy and convenient, and carbon compounds are so numerous that it is generally possible to choose analogous substances in order to find out whether the method is applicable in any particular case. This control can seldom be obtained in dealing with inorganic substances: if we wish to investigate thoroughly their molecular properties, we are bound to face the difficulties involved in the determination of their vapour-densities—difficulties which are especially great at high temperatures. It is true that the information thus obtained can be considerably extended by employing the freezing-point and boiling-point methods, as for example in the case of sulphur. Here the vapour-density method only proved with certainty the existence of molecules S_2, although it indicated that more complicated molecules might be present. The presence of such molecules—S_8—was proved by the freezing-point method; and by calculation from the experimental results, the temperature corresponding to the grade of dissociation S_6 was deduced.

The object of the present investigation is three-fold. In the first place, the author has endeavoured to obtain higher temperatures than have hitherto been used for vapour-density determinations of solid substances. Hitherto the highest temperature has been that obtained by V. Meyer and the author—between 1600° and 1700°,—using the Nilson-Pettersson method. A gas-furnace with an air-blast was used: porcelain vessels began to soften, and could only be protected from deformation by a platinum jacket. To attain higher temperatures the air-blast must be warmed and a more resistant material used for the vessels. In the present investigation, an improved Perrot furnace with hot air-blast was used. In this nickel melted with ease, and platinum-wire of 0·6 mm. diam. softened and became elongated under its own weight; thinner platinum-wire melted entirely. The melting-point of platinum has not been reached in any previous experiments of this kind. It lies about 1730°; so that it is certain that a temperature of about 1700° is obtained in this furnace. The only vessels which will stand this temperature without deformation are those made from a special body (for use as pyrometers in the *Reichsanstalt*) by the Royal Porcelain Factory. Vapour-density determinations of metals with this apparatus gave the following results:—

Arsenic: 5·30; 5·54; 5·39. Theoretical V.D. for $As_1=2·60$; for $As_2=5·20$.
Thallium: 14·77. Theoretical V.D. for $Tl_1 = 7·05$; for $Tl_2 =14·11$.
Cadmium: 4·34; 4·38. Theoretical V.D. for $Cd_1=3·87$.
Zinc: 2·64. Theoretical V.D. for $Zn_1=2·25$.

Neither indium nor tin could be vaporised, although, according to Carnelley and Carleton Williams, the latter boils at 1450°–1600°.

The second part of the investigation is devoted to arsenic trioxide, which is one of the few compounds whose molecular volume is not the simplest corresponding to its percentage composition. Mitscherlich (by Dumas' method) showed that its V.D. at 570° is 13·85,—the formula As_4O_6 requiring a V.D. of 13·68. This has been accepted as the molecular formula of arsenic trioxide, especially as V. and C. Meyer obtained similar results at a temperature which they estimated as 1560°. As a matter of fact, this temperature was incorrectly estimated and must have been nearer to 1050° or 1150°. The determinations now made at eight different temperatures show that the vapour-density (in presence of nitrogen) gradually decreases from 13·7 at 500° to 7·3 at 1730°. By extrapolation 1770° is deduced as the temperature at which only the simpler molecules As_2O_3 would be present. From his cryoscopic experiments Raoult concluded that arsenic trioxide does not dissolve unchanged in water, but forms ortho-arsenic acid. For the molecular weight he found the values 117·4 and 132·2, whereas the formula H_3AsO_3 would require the value 126. The author has confirmed these results (for vitreous and crystalline arsenic trioxide) by the boiling-point method. He has also used nitro-benzene as a solvent, and finds from its elevation of boiling-point the value 415 as the molecular weight of arsenic trioxide. The formula As_4O_6 gives a molecular weight of 396. Thus below 800° the molecules of arsenic trioxide have the formula As_4O_6; above this temperature, dissociation sets in.

The third part of the investigation is an attempt to discover suitable conditions for determining the vapour-densities of the alkali-metals. Little reliance can be placed upon previous determinations (by Dewar, Dittmar, and Scott) in iron and platinum vessels; for V. Meyer has shown that potassium and sodium cannot be vaporised in glass, platinum, silver, or porcelain vessels without attacking them. The problem is an interesting one, for researches on the alloys and amalgams of the alkali-metals (by Ramsay, Tammann, and others) point to monatomic molecules. The most promising type of vessel seemed to be a porcelain vessel glazed outside to prevent diffusion and covered on the inside with a protecting layer of carbon. The author has succeeded in depositing such a layer (0·5–1 mm. thick) by heating the porcelain vessel to a red-heat and passing through it coal-gas saturated with benzene. But experiments with potassium and sodium failed to give concordant results, probably because the metallic vapours acted upon the carbon coating or upon the gases occluded in it. D. E. J.

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414. *Equilibrium of a Stressed Solid in its Liquid, and Depression of its Melting-Point under Longitudinal Stress.* **E. Riecke.** (Wied. Ann. 54. pp. 731–738, 1895.)—ω_1, ω_2 being the specific volumes of a stressed solid and its liquid which produces that part of the stress which is represented by a hydrostatic pressure p, μ_1 and μ_2 their thermodynamic potentials, we find

$$\mu + p\omega_1 = \mu_2;$$

and if the strain is homogeneous, Ξ, H, Z tensions that with p make up the whole stress, m_1, ϵ_1, η_1 and m_2, ϵ_2, η_2 the masses, energy, and entropy of the solid and liquid, we have also

$$\epsilon_1 - \mathrm{JT}\eta_1 = \mu_1 m_1 = (\mu_2 - p\omega_1)m_1,$$
$$\mathrm{J}(\eta_2/m_2 - \eta_1/m_1)d\mathrm{T} - (\omega_2 - \omega_1)dp + \omega_1(\Xi d\xi + \mathrm{H}d\eta + \mathrm{Z}d\zeta) = 0,$$

ξ, η, ζ being the principal strains. Thus, for a constant hydrostatic pressure together with a longitudinal stress,

$$\mathrm{J}(\mathrm{Q}/\mathrm{T})d\mathrm{T} + \Xi\omega_1 d\xi = 0,$$

Q being the latent heat of fusion; and therefore the melting-point is lowered or raised according as $\Xi d\xi$ is positive or negative—*i. e.*, according as extension or contraction results with a tensional stress and as contraction or extension accompanies a pressural stress. If we replace ξ by $\{\Xi - (1-2\kappa)p\}/\mathrm{E} + \alpha(\mathrm{T} - \mathrm{T}_0)$, where E, κ are Young's modulus and Poisson's coefficient and α the thermal expansibility, this equation becomes

$$(\mathrm{JQ}/\mathrm{T} + \alpha\omega_1\Xi)d\mathrm{T} = -(\omega_1\Xi/\mathrm{E})d\Xi,$$

whence, putting $T_0, Q_0. \omega_0$ for T, Q, ω_1 (as their respective differences are small), and neglecting the value of $\alpha\omega_1\Xi$ in comparison with JQ/T, we get the approximate integral

$$T-T_0=-(\omega_0 T_0/2JQ_0E)\Xi^2.$$

For ice,

$$\omega_0=1{\cdot}09,\quad T_0=273,\quad Q_0=80,\quad E=2{\cdot}32\times 10^{10},\ \text{and}\ \ J=4{\cdot}2\times 10^7$$

in C.G.S. units: hence the coefficient of Ξ^2 is $1{\cdot}91\times 10^{-18}$; and if the stress is X kilograms' weight per sq. mm., then

$$\Xi=9{\cdot}81\times 10^7 X, \quad \text{or} \quad T-T_0=-{\cdot}0184 X^2.$$

The maximum value of X that can be used with ice is ·7, so that the maximum lowering of the melting-point is ·009° C. [In his calculation the author has neglected the divisor 2.]

A curious consequence of the above is that, if of two solid prisms within a solution from which they have been formed, one be subjected to longitudinal stress, it will melt, thereby increasing the concentration of the liquid and consequently causing deposition on the other prism which is not in equilibrium with the solution except at the lower concentration.

If $\Xi=H=Z=-\Pi$, so that in addition to the hydrostatic pressure of the liquid p the solid is acted on by hydrostatic pressure Π, then we have approximately

$$(JQ/T-3\alpha\omega_1\Pi)dT-(\omega_2-\omega)dp-(\omega_1-\omega)d\Pi = 0,$$

where ω is the value of ω_1 when $\Pi=0$. With $\Pi=0$ we have, without approximation, the well-known formula

$$dT/dp = T(\omega_2-\omega)/JQ.$$

R. E. B.

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415. *The Physical Changes attending the Tempering of Steel.* **G. Charpy.** (Bull. Soc. franç. Phys. No. 61, 1895.)—When steel which has been heated to a bright red is allowed to cool slowly, there occur at certain temperatures, called critical points, physico-chemical changes indicated by an evolution of heat. The author has investigated the nature of the changes which occur at these critical points, and the *rôle* these changes play in the alteration in the physical properties of steel produced by tempering. He finds that steel under the influence of rising temperature undergoes three distinct transformations, corresponding to three critical points (a_1, a_2, a_3). These critical points, which are quite distinct in the case of steel poor in carbon (about 700°, 740°, and 860° for almost pure iron), become merged in one for steel containing more than 0·5 per cent. of carbon.

The point a_1 corresponds to a transformation in the state of the carbon, which is characterized by an apparent diminution in the percentage of carbon as determined by Eggertz' calorimetrical method. It is to this transformation that must be attributed almost the whole of the changes in the mechanical properties of steel.

The point a_2 corresponds to a change in the iron, characterized by the disappearance of the flat place observed in the curves representing any deformation of the metal in terms of the applied stress (for instance, in the case of extension in the curve showing the connection between the elongation and the tension).

The point a_3 corresponds to a second change in the iron, which only appears to modify the magnetic properties of the metal.

W. W.

416. *Calculation of the Capillary Constants from Measurements of Drops of Moderate Dimensions.* **T. Lohnstein.** (Wied. Ann. 54. pp. 713–723, 1895.)—The paper is very mathematical. The fact is emphasized that a formula, originating from Poisson, viz.

$$r=\frac{a(1+\sqrt{2})}{4\sqrt{\pi}\sqrt{2}}\sqrt{\frac{a}{\mathrm{T}+a(\sqrt{2}-1)}}\,e^{\frac{\sqrt{\frac{1}{2}}}{a}(\mathrm{T}+a(\sqrt{\frac{1}{2}}-1))},$$

—where r is the radius of curvature at the summit of the drop, a has the meaning adopted by Quincke, and 2T is the maximum breadth of the drop—cannot be accurately applied to drops of small or moderate dimensions; as, for example, many of the drops mentioned in a paper by G. Meyer entitled "The Capillary Electrometer and Drop-Electrodes," which appeared in Wied. Ann. 53. 1894. In a particular case Meyer finds the value of the surface-tension to be 57·2 mg./mm. Lohnstein finds by one of his methods that the true value lies between 41·64 mg./mm. and 48·33 mg./mm.; by another method he finds that it is approximately 43·68, *i. e.* it differs from Meyer's value by 31 per cent.

A. Gs.

417. *Isomorphous Mixtures.* **F. W. Küster.** (Zschr. phys. Chem. 16. pp. 525–528, 1895.)—The article strongly opposes the view, expressed (according to the author) by H. Ambronn and Le Blanc in Ber. d. math.-physik. Klasse der königl. Sächs. Ges. der Wiss. zu Leipzig, 2 Juli, 1894, that crystals composed of isomorphous bodies may be looked upon as mere mixtures (*Gemengen*), and that there is no intimate mingling (*Mischung*) or molecular interpenetration.

A. Gs.

418. *Original Air-Pump of Otto von Guericke.* **E. Berthold.** (Wied. Ann. 54. pp. 724–726, 1895.)—The instrument referred to is the original of the third type of Guericke's air-pumps. The author proves that the air-pump belonging to the Königlichen Bibliothek of Berlin is *not* the original. Johann Wimmerstedt states that the noted Dr. Heraeus took the original with him to Sweden in 1676; in 1734 it was in the care of the Professor of Mathematics of Lund, but belonged to Werner. What became of it afterwards is not known.

A. Gs.

419. *Dilatation of Water.* **S. de Lannoy.** (C. R. 120. pp. 866–868, 1895.)—Previous measurements of the expansion of water are rendered somewhat unreliable on account of the difficulty of ascertaining the temperature to within 0°·1 C. for temperatures under 50° C. and 0°·2 for higher temperatures. In the present experiments the apparent expansion of water is measured in a glass dilatometer of about 5 c.c. capacity, immersed in a large water-bath along with a thermometer carefully compared with standards. The principal causes of error are:—(1) the volume of the dilatometer is only known to within 1 part in 5000; (2) the coefficient of expansion of glass may differ by 0·000001 from the value taken; (3) compressibility of water and evaporation from the surface of the liquid in the dilatometer are neglected; (4) there is a possible difference of 0°·03 to 0°·08 C. between the temperatures of the water in the dilatometer and that in the bath surrounding it. The results obtained differ very slightly from those of Rosetti, thus indicating the reliability of the method used. The author proposes to examine by the same apparatus the expansion of saline solutions.
J. L. H.

420. *Internal Friction of Metals.* **M. Cantone.** (N. Cim. 4. 1. pp. 205–219, 1895.)—The experiments described in an earlier paper (*ante*, p. 246, Abstract No. 411) have been continued for other metals. The results are generally similar to those for brass and iron. Nickel does not depart much from Hooke's law; its plasticity (as measured by the loss of energy in a cycle) diminishes slightly by accommodation, but shows a marked increase when the wire is subjected to a large strain. Platinum for small strains showed some of the characters of iron, especially that the plasticity was greater by the static method than the kinetic; but when strained more, it approached the simpler behaviour of the soft metals. Aluminium is highly plastic, and departs much from Hooke's law; the static and kinetic methods show considerable divergences. Copper and silver show the most striking agreement between the plasticity measured by the two methods—*i. e.*, the damping of vibrations is clearly due to static hysteresis; and a change in the time of vibration makes no difference to the energy lost per vibration, although those metals show much elastic after-effect: accommodation causes but a slight decrease in the damping. The author gives a method for soldering aluminium. R. A. L.

421. *Temperature Minima of this Winter on Mont Blanc.* **J. Janssen.** (C. R. 120. pp. 807–809, 1895.)—The thermometers, made by Tonnelot, have been fixed horizontally in cupboards fitted with wire gauze in front, at the top, and bottom, and provided with chimneys which help to maintain the same temperatures within and without the cupboards. To determine the temperatures at different depths of the snow-caps, holes were made in a thick board of 2·50 m. length, and thermometers placed horizontally in these

holes, which were covered up with sheet-iron. These snow-temperatures have not yet been ascertained. The minima observed on the summits were:—Mont Brévent, −26°; Mont Biret, −33°; Mont Blanc, −43°. In the Jura mountains the temperature has also fallen below −30°; so that the cold wave of this winter has extended to the mountain-tops. The meteorograph to be placed on the Mont Blanc observatory during this summer will permit of continuous observations. An equatorial, partly the gift of Henry (Frères), will also be mounted in the Mont Blanc observatory. H. B.

422. *Downward Non-Gyratory Air-Currents in Waterspouts and Tornados.* **H. Faye.** (C. R. 120. pp. 851–857, 1895.)—The author regards waterspouts, cyclones, and typhoons as phenomena of identical and not only of analogous type, differing in magnitude only. The mechanical analogue is furnished by an experiment which G. A. Hirn published in 1878. When gyrating water-currents are produced in a vessel by water entering at the sides, a cone is formed full of water with a slight depression at the surface. When now an opening is made in the bottom of the vessel, the water rushes out, and air descends down the funnel in such a way that a stick held in this cone becomes fixed without disturbing the flow. Hirn ascribed the downward movement of the air to some unknown electrical effect. The author considers the phenomena as purely mechanical. From a cloud moving rapidly over the ground, gyrating cones project with their apices turned downward. As long as these cones do not strike the ground, no noise is heard and no damage done. When they strike, however, the air within these cones escapes in all directions, with destructive effects. When the gyration decreases, the cones recede; but recrudescences occur. In a waterspout a cloud-shell in violent gyration surrounds the air-cone like a funnel-shaped sack; the air inside is nebulous, charged with moisture, and only slightly in gyration. When the cone meets with an obstacle, the air rushes out below, and a downward current, without gyration, is produced. As a rule, tornados and waterspouts are too opaque to show the funnel with its shell; occasionally, however, the four parallel borders of the tubular end of the funnel can be distinguished. The author quotes two cases in confirmation of his theory. The 'Eglé,' at anchor in the Mozambique Channel, experienced a rain-storm on April 1st, 1858. The barometer fell from 758 to 742 and 740 mm. At 11 P.M. the violent south-easter suddenly stopped, the sky became serene, and the air absolutely calm; at 1 A.M. the rain set in again, with a strong north-wester, which wrecked the ship. The calm zone had a diameter of 18 km. The second, still more characteristic case, taken from Sprung's 'Meteorologie,' concerns a typhoon which passed over Manilla on October 20th, 1882. The registering instruments of the observatory recorded velocity and direction of the wind, temperature, and humidity. The temperature sank to 77° F. during

the storm, rose to 85° and remained there all through the period of calm, and fell again abruptly to 77° afterwards. The humidity was 90° whilst the tornado was blowing, fell to 36° (an abnormally low value for the district) during the calm period, and rose again to 90°. The calm zone, outside which the storm raged, had a diameter of 21 km. According to Faye, the whole disturbance descends, drawing cirrus-clouds down with it, which supply the rain, but leaves a partial vacuum into which the clouds do not enter, which is filled up again by drier air. This air follows the advancing storm without gyrating. H. B.

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423. *Gravitation in Russia.* **G. Defforges.** (C. R. 120. pp. 909–911, 1895.)—In conjunction with Wittram of Poulkowo the author has determined, with the aid of his apparatus, the relative intensities of gravitation at Poulkowo, Tiflis, Ouzoun Ada, Bukhara, Taschkend. The positive anomaly characteristic of the seashore, and the continental negative anomaly were well observed. The positive anomaly of +8 (9·8201 against 9·8193 calculated) at Poulkowo is twice as large as that of the Mediterranean, and four times as large as that of the North Sea, all compared to Paris. Taschkend (400 m. above sea-level) gave −18, Ouzoun Ada on the Caspian Sea −12, Bukhara 0,—that is to say, at Bukhara the anomaly was of the same order as at Paris. H. B.

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424. *Formation of Crystals at the Bottom of Solutions of Higher Specific Gravity.* **L. de Boisbaudran.** (C. R. 120. pp. 859–860, 1895.)—After having established (these Abstracts, p. 55) that crystals can aggregate near the surface of solutions of lesser density, the author demonstrates the inverse effect. By varying the temperature, crystals floating in saturated solutions can be made to redissolve and to recrystallise at the bottom of the vessel. Sodium sulphate + 10 aq. is dissolved in a saturated solution of iodide of sodium, which has a greater density than the hydrated sulphate. The crystals of sulphate first aggregating on the surface disappear gradually, and gather after a few days round a sulphate crystal previously fixed at the bottom of the test-tube. A lump of ice of 1 or 2 gr. remained unchanged in a weak aqueous solution of ammonia, on which it floated, as long as the temperature was kept at −3°·5. When the temperature was varied between −3°·25 and −3°·75 for six hours or more, the ice re-formed in the lower part of the test-tube. H. B.

LIGHT.

425. *New Method of deducing the General Formulæ for the Diffraction Phenomena of Fraunhofer and Fresnel, by Apertures on Curved Surfaces (determined for the first time by H. Nagaoka); New Application of the first of these Formulæ to the case of an Orifice made in a Sphere.* **M. Cinelli.** (N. Cim. pp. 141–155, 1895.)—The intensity of light at any point A distant r from a point P in the aperture is given by either

$$\text{(1)}\quad \mathrm{I}=\mathrm{I}_1\left\{\frac{1}{2}\left(\frac{dr_1}{dn}-\frac{dr}{dn}\right)\frac{1}{\lambda}\frac{r}{1}\right\}^2\left\{\mathrm{C}^2+\mathrm{S}^2\right\},$$

or

$$\text{(2)}\quad \mathrm{I}=\mathrm{I}_1\left\{\frac{1}{\lambda r}\frac{dr}{dn}\right\}^2\left\{\mathrm{C}^2+\mathrm{S}^2\right\},$$

the first corresponding to the phenomena of Fraunhofer, the second to those of Fresnel.

In these expressions r_1 is the distance of the point P from the source of light; λ is the wave-length;

$$\mathrm{C}=\int_s \cos\frac{2\pi}{\lambda}(r+r_1-\alpha)ds=\int_s \cos\theta ds,$$

$$\mathrm{S}=\int_s \sin\frac{2\pi}{\lambda}(r+r_1-\alpha)ds=\int_s \sin\theta ds,$$

the integration being over the whole aperture supposed plane; and I_1 is a function of r_1 and the direction of the light.

For a plane aperture, take its plane for that of $x\,y$. Let $x_1\,y_1\,z_1$ be the coordinates of C the source of light, $x\,y\,z$ those of A the point at which the intensity is considered, $\xi\,\eta\,\zeta$ (ζ being in this case zero) those of a point P in the aperture. Also let $\rho\,\rho_1$ be the distances from the centre of the aperture to A and C respectively. Then it is found that

$$\theta=\frac{2\pi}{\lambda}\left[\xi(\alpha_1-\alpha)+\eta(\beta_1-\beta)\right];$$

in which

$$-\alpha=\frac{x}{\rho},\quad -\beta=\frac{y}{\rho},$$

$$\alpha_1=\frac{x_1}{\rho},\qquad \beta_1=\frac{y_1}{\rho}.$$

If the aperture be taken in a curved surface instead of a plane, it is found that

$$\theta=\frac{2\pi}{\lambda}\left\{\xi\left(\frac{x}{\rho}+\frac{x_1}{\rho_1}\right)+\eta\left(\frac{y}{\rho}+\frac{y_1}{\rho_1}\right)+\zeta\left(\frac{z}{\rho}+\frac{z_1}{\rho_1}\right)\right.$$
$$+\frac{1}{2\rho^3}(x\xi+y\eta+z\zeta)^2$$
$$\left.+\frac{1}{2\rho_1{}^3}(x_1\xi+y_1\eta+z_1\zeta)^2-\tfrac{1}{2}(\xi^2+\eta^2+\zeta^2)\left(\frac{1}{\rho}+\frac{1}{\rho_1}\right)\right\}.$$

In Fraunhofer's case $\frac{1}{\rho}, \frac{1}{\rho_1}$ are supposed infinite, as are also $x\,y\,z$, $x_1\,y_1\,z_1$; and this expression reduces to the form

$$\theta=\frac{2\pi}{\lambda}\{\xi\overline{\alpha_1-\alpha}+\eta\overline{\beta_1-\beta}+\zeta\overline{\gamma_1-\gamma}\};$$

whence, writing

$$\frac{2\pi}{\lambda}\overline{\alpha_1-\alpha}=l,$$
$$\frac{2\pi}{\lambda}\overline{\beta_1-\beta}=m,$$
$$\frac{2\pi}{\lambda}\overline{\gamma_1-\gamma}=n,$$

we get

$$\mathrm{I}=\mathrm{Mod.}^2\int_6 \epsilon^{i(l\xi+m\eta+n\zeta)}ds, \quad . \quad . \quad . \quad . \quad . \quad \text{(A)}$$

as was obtained by Nagaoka. In Fresnel's case we get

$$\mathrm{I}=\mathrm{Mod.}^2\int_s \epsilon^{i\frac{\pi}{\lambda}\left[(\alpha\xi+\beta\eta+\gamma\zeta)^2-(\xi^2+\eta^2+\zeta^2)\right]\left(\frac{1}{\rho_1}+\frac{1}{\rho}\right)}ds, \quad . \quad . \quad \text{(B)}$$

also obtained by Nagaoka.

Our author next considers a particular application of the formula A, namely, to the case of an aperture (*feuditura*) made on a sphere, bounded by two meridians very near together and two parallels, however situated. The mathematical treatment of this case does not admit of representation in an abstract. The calculated results are said to agree with experiment satisfactorily.

S. H. B.

426. *Rotativity of Bodies in the Crystalline State and in Solution.* **H. Traube.** (Sitz. Akad. Wiss. Berlin, pp. 195–205, 1895.)—Patchouli camphor and ordinary camphor have the same molecular rotation when in crystals and when dissolved. Matico camphor and rubidium tartrate in crystals have greater rotativity than when dissolved. The dextrotartrate crystals rotate to the left and the lævotartrate to the right. Crystallographic details are given of the substances. The author concludes that the cause of the rotativity of the dissolved substance, viz., the arrangement of the atoms in the molecule, is still present in the solid, and superposed on this there is in some cases a rotativity due to the arrangement of the molecules in the crystal.

J. W. C.

427. *Radiometer of Symmetrical Construction.* **G. Seguy.** (C. R. 120. p. 725, 1895.)—If the radiometer is exposed, in the usual way, to light the vanes remain at rest. They rotate with energy if the instrument be put in a box, provided with two side-windows in such positions that the rays of light strike the vanes tangentially to the circle produced by their rotation, and in opposite directions like the forces of a couple. A. Gs.

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428. *Fluorescence of Solutions.* **O. Knoblauch.** (Wied. Ann. 54. pp. 193-220, 1895.)—The intensity of the fluorescence of a solution is known to depend upon the dissolved substance, and also upon the concentration of the solution and the nature of the solvent. But quantitative investigations of this relation have hitherto been almost entirely lacking. The experimental work here described was undertaken to supply this deficiency. In the photometric comparison of the fluorescence of different solutions, a Linnemann's zircon burner is used as the luminous source. Its light, after concentration by a lens, falls at an angle of 45° upon a plane surface of the solution under examination. The collimator of a Glan's photometer is placed normal to the same surface, and thus receives only the light due to fluorescence and not that due to reflection at the liquid surface. The fluorescent light enters the upper half of the collimator, the lower half being illuminated by light from the same source, but reflected respectively by a paper screen and a totally-reflecting prism. The method of measurement here employed is based upon the assumption that the intensity of the fluorescent light is proportional to that of the exciting light. In order to ascertain between what limits this holds good, a preliminary experiment was made as follows. A smoked glass, which transmits only 1/6400th of the light incident upon it, is held (1) between the luminous source (sunlight) and the fluorescent solution, and (2) between the solution and the photometer. The readings of the photometer in the two cases are found to be in accord. From this the author infers that "The intensity of the fluorescent light is proportional to that of the exciting light even when the latter is changed in the ratio 1 : 6400." In the photometric observations over 80 solutions are dealt with, 13 different substances being used, each dissolved in from 2 to 12 different solvents. It is noteworthy that the changes in the intensity of the fluorescent light due to changes of solvents only differ greatly in magnitude and even in sense also where different dissolved substances are concerned. Thus the fluorescent constants for fluorescein dissolved respectively in amyl-alcohol and in methyl-alcohol are in the ratio 3 : 1. Whereas lithium-fluorescein in the same solvents, and taken in the same order, furnishes values in the ratio 1 : 5.—In the second part of the paper the author advances a theory of fluorescence based upon the electromagnetic theory of light and the theory of electrolytic dissociation. This theory is confirmed by the experimental results previously named, which make it impossible to place the solvents in any one order

such that for *any* the *same* dissolved substance the fluorescence of the resulting solutions for like concentrations shall always increase as one passes along the series of solvents. E. H. B.

429. *Minimum Temperature of Visibility.* **P. Pettinelli.** (N. Cim. 4. 1. pp. 183–186, 1895.)—Experiments made by heating a cylinder in the dark to 460°, and allowing it to cool, showed that (i) a large surface is visible at a lower temperature than a small one (by 60°); (ii) lampblack is visible about 20° lower than metals: the mantle of an Auer lamp showed about the same power of radiation as lampblack; (iii) resting the eye makes no appreciable difference: different eyes varied by about 6°. The average minimum temperature of visibility for a large black surface was 404°.
R. A. L.

430. *The Ketteler-Helmholtz Dispersion-Formula.* **H. Rubens.** (Wied. Ann. 54. pp. 476–485, 1895.)—The dispersion of fluorite has been measured by the author and by Paschen (Wied. Ann. 53. p. 301); but as the author had only a wire grating, he concludes that Paschen's measurements carried out with one of Rowland's gratings are to be preferred; and shows that the spectra of a wire-grating disagree somewhat with the ordinary theory. He has, therefore, recalculated the dispersions of rock-salt, sylvin, quartz, and flint-glass, which were obtained by comparison with that of fluorite; using for quartz and flint-glass the complete formula

$$n^2 = b^2 + \frac{M_1}{\lambda^2 - \lambda_1^2} - \frac{M_2}{\lambda_2^2 - \lambda^2},$$

and for the other two the simplified form

$$n^2 = a^2 + M_1/(\lambda^2 - \lambda_1^2) - k\lambda^2.$$

The constants are :—

	b^2.	M_1.	λ_1^2.	M_2.	λ_2^2.
Quartz............	3·4629	0·010654	0·010627	111·47	100·77
Heavy silicate flint (0,500).	6·7716	0·03672	0·0404	1508·2	394·65

	a^2.	M_1.	λ_1^2.	k.
Sylvin	2·1738	0·0150	0·0234	0·000599
Rock-salt	2·3285	0·018496	0·01621	0·000920

The curves are not given, as they are hardly distinguishable from the former ones. R. A. L.

431. *Astigmatism of Lenses.* **G. Foussereau.** (J. de Phys. 4. pp. 169–178, 1895.)—The author calculates the loci of images formed by oblique pencils of light passing through the optical centre of a lens or mirror; the lens is supposed thin, and quantities below the first order neglected. The chief result of interest is that the image of a plane at right angles to the axis of the lens

is an ellipsoid or hyperboloid of revolution, whose radius of curvature at the vertex is half the focal length of the lens. The second nodal point of the eye is close to the inner face of the crystalline lens: hence rays pass centrally through that face; and it is suggested that the curvature of the central spot of the retina may be an adaptation to the shape of the image. R. A. L.

432. *Luminescence of Salts.* **E. Wiedemann** and **G. C. Schmidt.** (Wied. Ann. 54. pp. 604–625, 1895.)—The authors find that many substances, after being exposed to the kathode rays, have the power of emitting light when warmed (Thermoluminescence), and some glow when thrown into water (Lyoluminescence), or rubbed (Triboluminescence). To produce strong kathode rays an electrode sealed into a tube is connected with the + pole of a 20-plate induction-machine, and a wire connected with the − pole is brought near the outside of the tube. When the terminals of the machine are 1 cm. apart, a powerful stream of kathode rays is produced in the tube, even at pressures of several mm. To determine whether the action is due to a chemical change or merely to physical change, NaCl was investigated fully. After exposure to the negative rays, the salt turns brown. On heating, it at first goes blue, and then gradually becomes white. So long as any blue colour remains, the salt shows thermoluminescence. If it is dissolved in water, the solution is colourless and has an alkaline reaction. The authors ascribe these effects to the formation of Na_2Cl by the kathode rays. The removal of the colour on heating they suppose due to the formation of NaOH and NaCl by the action of water driven from the glass. This explanation is supported by their observation that after repeated exposure to the kathode rays and removal of the colour, the salt becomes less sensitive to the action of the rays, on account of the increasing proportion of NaOH present. Electrolytically prepared K_2Cl shows thermoluminescence similar to that shown by KCl that has been exposed to the negative rays. Hence the authors conclude that Goldstein's explanation of the phenomena (Abstract No. 320) is wrong. Similar luminescence is shown by the sulphides and sulphates of the alkaline earths and the zinc group, and in a high degree by "solid solutions" of a small quantity of one salt in another, formed by the simultaneous precipitation of the two salts or by other methods described in the paper. A calcium salt containing 1% of a manganese salt is especially effective. J. W. C.

433. *Historical Note on the Effect of the Earth's Motion on Refraction-phenomena.* **A. Leduc.** (J. de Phys. 4. pp. 106–109, March 1895).—Arago's conclusions, based on the emission theory of light, follow from the wave-theory with Doppler's principle, and the negative result of his comparison of the real altitudes of stars that are being approached or receded from with their apparent altitudes when viewed through a prism with horizontal edge is due to his using an *achromatised* prism. The conclusions might

best be verified by a comparison of the most refrangible parts of the spectra of direct sunlight and of sunlight reflected from a planet when the earth and planet are both approaching and receding from each other with their maximum relative velocity. R. E. B.

434. *Fluorescence of Argon charged with Benzene Vapour.* **M. Berthelot.** (C. R. 120. pp. 797–800, 1895.)—Argon at atmospheric pressure charged with benzene vapour and traversed by the silent discharge shrinks in volume, presumably from the formation of a compound of argon and benzene. When the discharge has passed for a period of from 15 min. to 4 or 5 hours, the tube becomes lighted up throughout its extent with a violet light. The light gradually grows brighter till it reaches a brilliant green, visible in full daylight. The glow disappears as soon as the discharge is stopped, and reappears at once on starting the current again, if the stoppage has been short. The spectrum of the glow shows a series of sharp bands, whose wave-lengths were determined approximately with a single-prism spectroscope, the light being too faint to allow higher dispersion to be used. The brightest are the four following. A yellow band at 579, marked with fine absorption-bands; a green band at 547, also marked with fine black lines; two violet bands at 438 and 436. These do not coincide with the helium line, or the chief auroral line, but they correspond to the brightest lines in the argon spectrum.

J. W. C.

435. *Photography in Natural Colours by the Indirect Method.* **A.** and **L. Lumière.** (C. R. 120. pp. 875–876, 1895.)—The method suggested by MM. Cros and Ducos du Hauron has scarcely received any practical application hitherto, on account of difficulties experienced in the choice of colours and in securing and superposing the monochromes. The authors have studied these points. They have found it expedient to retain the use of coloured screens (orange, green, and violet), and have prepared three series of photographic plates having respectively a maximum sensitiveness for the rays transmitted through one of the screens. The method adopted for printing the monochromes depends upon the use of bichromated gelatine. This, by itself, does not give satisfactory photographic images with half-shadows; but it acquires this property when certain precipitates, such as silver bromide, are added to it. To a 10 per cent. solution of gelatine are added 5 p. c. of ammonium bichromate and 5 to 10 p. c. of silver bromide in emulsion; and a glass plate is coated with a very thin layer of this preparation. The film is exposed under the negative to be reproduced, and then washed with cold water. The parts which have been rendered insoluble remain as a barely-visible image, which can be coloured. The silver bromide is removed by a suitable solvent (*e. g.* hyposulphite). We can thus obtain with ease images of any colour possessing all the gradations of the negative. Polychromic prints are built up by the successive

preparation, on the same plate, of three monochromic images (*e. g.*, red, yellow, and blue images, from corresponding negatives). Each image must be isolated from the one below by an insoluble layer, *e. g.* of collodion. Modifications can be effected by varying the depth of colour in one of the images or by superposing a fourth image. The superposed images can be transferred all together to paper. D. E. J.

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436. *The Spectra of the Planets.* **H. C. Vogel.** (Sitz. Akad. Wiss. Berlin, pp. 5-25, 1895.)—In this paper the author has collected and discusses the results obtained from observations of planetary spectra by himself, Huggins, and others. It is pointed out that in such observations there is no very great advantage in using telescopes of large aperture, because the brightness of the image depends on the ratio of aperture to focal length, and this ratio may be the same for a large telescope as for a smaller one. In addition, the greater thickness of the lenses in large telescopes causes a reduction in the brightness of the more refrangible parts of the spectrum. If the image of the planet is too small to cover the slit of the spectroscope, the greater width of the spectrum obtained by using a large telescope renders observation and comparison of lines more easy; and in cases where a cylindrical lens must be used with small instruments, there is an advantage in increased aperture, but such advantage is not nearly as great as it would be in the case of observations on fixed stars. Consequently, neither the introduction of large telescopes, nor the application of photography to this class of work, has resulted in anything more than a confirmation of previous results. The comparative dispersion in the various observations quoted in the paper may be gathered from the following figures, which represent the distance in millimetres between the Fraunhofer line F and a point midway between H and K:—Huggins, 5·3; Vogel: 7·0 for Jupiter, Saturn, and Uranus, 16·0 for Mars and Venus, and 69·0 for several observations on Venus.

Mercury. Three photographs of the spectrum of this planet were taken consecutively on the same evening. In one of these the spectrum of the surrounding sky is also visible, and the range is from λ 487 $\mu\mu$ to λ 380 $\mu\mu$. The spectra are exactly coincident, and 28 lines can be identified. In the other two photographs the range is smaller, extending only to K; 15 lines can be distinguished. On another occasion, a photograph of the planet and surrounding sky showed 16 coincident lines.

Venus. In the spectrum of Venus over 500 lines were identified with those of Rowland's map of the solar spectrum in the region between λ 406 $\mu\mu$ and λ 460 $\mu\mu$; the relative intensities being the same in both spectra. In 1879 Huggins obtained photographs of the spectrum of the planet and the surrounding sky, and identified 80 lines between λ 320 $\mu\mu$ and λ 480 $\mu\mu$.

Mars. Between F and K 75 lines were recognized as belonging to the solar spectrum, and no deviations of any kind were detected in this region. Wilsing obtained similar results with three successful exposures; and Huggins writes that in November of last year he found complete identity of the ultra-violet spectra of Mars and the Sun. These results dispose of the theory that the red colour of the planet is due to groups of lines in the blue and violet. In 1877 Maunder compared the spectra of Mars and the moon, when the two bodies had nearly equal altitudes; the altitude was however only about 26°, and there was considerable atmospheric absorption. Some lines in the spectrum of Mars appeared broader and more conspicuous than in the Moon's spectrum; also variations were found in the spectrum from different parts of the planet's surface, causing alterations in the relative intensity of whole regions of the spectrum. Campbell, after observations on Mars and the Moon in high altitudes at Lick Observatory last year, concludes that the spectroscope fails to detect any atmosphere on the planet. The author, and afterwards Scheiner and Wilsing, working with a lens of aperture $\frac{1}{10}$ of its focal length, found that the telluric lines are more prominent in the spectrum of Mars than in that of the Moon; and Huggins has noticed an absorption-band in the planet's spectrum, not present in sunlight, very slightly to the more refrangible side of the D line. The evidence concerning the existence of an atmosphere round the planet is therefore of a very conflicting character.

Jupiter. In the best photograph 100 lines were identified as belonging to the solar spectrum. The two equatorial bands give rise to greater absorption, but the difference is one of degree, not of kind; the extra absorption is most marked near the F line. Referring to Draper's observation that the red spot on the planet is a source of light, the author states that Copeland and Lohse, observing at Dun Echt and Potsdam respectively on the same day as Draper, found the spectrum of the red spot to be analogous in character to that of the equatorial bands. He considers Draper's result to be due to some defect in the photographic film used. Spectra of the satellites of Jupiter show as many as 40 Fraunhofer lines.

Saturn. The spectrum shows 30 Fraunhofer lines, and contains no trace of deviation from that of sunlight. Huggins obtained a spectrum with over 20 lines more refrangible than K, and coincident with lines from the surrounding sky photographed on the same plate. The rings give exactly the same spectrum as the planet, excepting that Keeler found an absorption-band $\lambda\, 618\ \mu\mu$, characteristic of the planet's spectrum, to be absent from that of the rings. The rings are also much brighter than the planet.

Uranus. Observations at Potsdam by Frost, at Lick by Keeler, and in London by Huggins, all agree in showing that the lines in the spectrum of Uranus are Fraunhofer lines. There is no evidence of any absorption-bands or bright lines in the more refrangible part of the spectrum: consequently Lockyer's theory that the planet is self-luminous cannot be upheld. Keeler observed the spectra of the two outer satellites of Uranus. J. L. H.

HEAT.

437. *Specific Heat and Boiling-Point of Carbon.* **J. Violle.** (C. R. 120. pp. 868–869, 1895.)—The specific heat of carbon was determined by means of a fragment of graphite heated in a suitable electric furnace. The temperature of the part of the furnace used was measured either calorimetrically, by a piece of platinum or iridium, or optically, by observing the intensity of the light emitted from the graphite. The results show that at temperatures above 1000° C. the mean specific heat of graphite increases linearly with temperature, according to the formula $C=0{\cdot}355+0{\cdot}00006\,t$. The heat evolved from a gramme of solid graphite in cooling from its point of volatilisation to 0° C. is 2050 calories; hence the boiling-point of carbon is 3600° C. J. L. H.

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438. *Verification of Clapeyron's Fusion Formula.* **De Visser.** (Rec. trav. chim. Pays-Bas, 11. pp. 100–140.) Considering all previous experimental verifications as only *qualitative*, the author experimented with pure acetic acid, the specific volume of which, as liquid, is ·94953, at the normal fusing-point 16°·5965 C. He determined dT/dp by the *manocryometer*, which is practically Bunsen's apparatus, wherein the change of pressure is caused by the change of state of the substance, and not by the expansion of mercury, and found ·02425 between 1 and 12 atmos and ·02401 between 12 and 26 atmos. He further determined $(v_1-v_2)/L$ by introducing warm water into the test-tube of a Bunsen's calorimeter, in which water and ice were replaced by liquid and solid acetic acid, and found in the mean ·0000034425, which value was verified by direct estimation of v_2-v_1 by a dilatometer, and of L by determination of the heats of solution of solid and liquid acetic acid in water. Putting this last value in Clapeyron's formula we get $dT/dp=\cdot 02421$, which agrees well with the direct determination. A column of solid acetic acid enclosed with a little air at either end in a sealed capillary tube with thick walls is useful for lecture-purposes, as it may be raised far above melting-point without changing state. R. E. B.

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439. *New Method of Measuring Temperature.* **D. Berthelot.** (C. R. 120. pp. 831–834, 1895.)—Since the laws obeyed by gases are simpler than those obeyed by liquids and solids, the author seeks a method of thermometry depending on the properties of a gas alone, and independent of the enclosing vessel. $\frac{\mu-1}{d}$ is known to be constant for gases, and hence if the pressure of a gas is known, the temperature can be inferred from the refractive index. Changes of μ are observed by an interference method. A ray of light is divided into two parts, and half is sent along each of two tubes containing the same gas at atmospheric pressure.

The parts are then re-combined so as to give interference-fringes. If, now, one of the tubes is heated, the pressure remaining constant, the fringes are displaced, but can be brought back by reducing the pressure in the cool tube. The density is now the same in the two tubes, and since the temperature and pressure in the cool tube are known, the temperature in the other can be calculated. A second method consists of bringing back the fringes by increasing the pressure in the hot tube, in which case the second tube can be dispensed with. Only the central part of the tube is heated, the ends being cooled to a definite temperature, and allowance is made for the part where the temperature is variable by making two experiments on tubes heated over different lengths, and taking the difference of the results. The method appears to be capable of measuring temperatures up to 200° to one or two tenths of a degree. The author proposes to apply the method to electric furnaces and the like. J. W. C.

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440. *Standard Thermometers.* **J. Pernet, W. Jaeger,** and **E. Gumlich.** (Zschr. f. Instrumk. 15. pp. 2–13, Jan. 1895; pp. 41–54, Feb.; 81–89, March; and 117–132, April 1895.)—This is a series of communications from the laboratory of the *Physikalisch-Technische Reichsanstalt* on the construction and testing of standard mercury thermometers. The first difficulty attending the use of the best thermometers is the unknown character of the influence of the composition of the glass upon the indications of the thermometer. It is not possible to predict the influence of the expansion of the glass even in the case of thermometers of identical composition, so long as the bulbs are blown from the stems. The volatilisation of some constituents in the course of this process, which gives rise to these uncertainties, may be avoided by making the bulbs of thin-walled tubes and welding them on to stems of the same composition. As regards residual thermal effects, it was found by Wiebe and Schott that these occur to the greatest extent in glasses containing considerable quantities of potassium and sodium together, but that they almost vanish when only potassium or only sodium is present. The present commercial supply of thermometer-glass with known composition and properties may be considered satisfactory, especially since the introduction of Schott's thermometer-glass No. 59^{III}, whose maximum depression of the freezing-point after heating to 100° does not exceed 0°·02. In deciding whether the depressed freezing-points or those attained after prolonged rest shall be reckoned from, Herr Pernet's observation must be borne in mind, according to which the use of the depressed freezing-points is attended by more constant fundamental intervals, and makes the measurements independent of the succession of temperatures.—In order to measure to, say, 0°·002, the length of a degree should not be less than 6 mm., and since lengths of stem beyond 60 cm. are very inconvenient, the range of each thermometer cannot well exceed 100°. Several thermometers constructed at the Reich -

anstalt contain the points 0°, 50°, 100°, 150°, 200°, and 250°, the intervals between each of these temperatures being taken up by bulbs containing about 50° of mercury, except the interval between 100° and 150°, which is divided into degrees in the ordinary manner. Some have only two auxiliary bulbs, below 100° and above 150° respectively, and their range is from 50° to 200°. These are divided in fifths or tenths of a degree, and they are only tested to 0°·005.—In order to reduce the thermometers tested to an ideal thermometer, with faultless division and perfectly cylindrical stem, always exposed to the same external and internal pressure and showing no kind of thermal after-effects, the following operations had to be performed :—

1. Testing the division.
2. Calibration of the stem-tube.
3. Determination of the changes due to variation of external pressure.
4. Determination of the dilatation of the thermometer-bulb due to pressure of the mercury-column at different temperatures.
5. Determination of the fixed points, and the value of the degree, with reference to the thermal after-effects.

As regards division, the principle of equal lengths was adopted, instead of that of equal volumes of mercury. The dividing-engine employed was provided with an excellent screw by Brauer, of St. Petersburg, 89 cm. long, with a pitch of about 1·01 mm. The progressive and periodic errors of this screw were first determined, and as they excluded an automatic division, the position of the screw for each of the 1100 lines was calculated beforehand, and adjusted to within 0·001 mm. by means of a vernier.—In calibrating the thermometers, intervals of 1° were taken, since intervals of 2°·5 were found too large for accurate interpolation. Threads of various lengths were divided off not by local heating, which may produce permanent changes of internal gauge, but by bringing the microscopic residual bubble of air or steam into the lower end of the stem—which is contracted for the purpose—when a light knock will liberate the thread. The thermometers were simple stem thermometers, without enamel backing or enclosing tube, so that the calibration could be carried out by two microscopes reading the scale from the face and through the back. The thread-lengths varied from 10° through 10° up to 90°, and the positions were read at intervals equal to the length of the thread. Finally, the corrections for every degree were ascertained by Gay-Lussac's method with a thread of 1° length. As a result, the sums of the errors of volume and division for each of the scale-divisions of the chief first-class standards, which show no widenings and no sudden irregularities of volume, may be taken as accurately determined to a thousandth of a degree.—The internal pressure consists of the hydrostatic component due to the weight of the mercury, and the capillary component due to the top of the mercury column. The latter is only constant when the mercury is steadily rising. It is quite incalculable when falling, and since it is also very small, it is safe and convenient to neglect it. The hydrostatic component is determined by inclining the thermometer at various angles to the horizon at various temperatures. The coefficient of external pressure is determined with an apparatus in which the thermometer can be exposed to various pressures ranging from atmospheric pressure to a vacuum.

In determining the true freezing-point, it was found most important to obtain pure ice from distilled water, and to obviate the difference of temperature between the surface and the centre existing in all machine-made ice by fine shaving and mixing with distilled water. The determination of the boiling-point requires an accurate knowledge of the pressure to within 0·02 mm., which implies a knowledge of the internal temperature of the mercury in the barometer, and an accurate correction for capillarity. A careful consideration of all these and other sources of error enables the authors to claim an accuracy of 0°·001 for their best thermometers. The coefficient of apparent expansion of mercury between 0° and 100° in the Jena glass marked 16^{III} was found to be 0·0001571. E. E. F.

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441. *Influence of Heat on the Strength of Manganese Bronze.* **M. Rudeloff.** (Mitth. kön. Tech. Versuchs-anstalten, Berlin, 13. pp. 29–42, 1895.)—In 1893 the author ascertained that the strength of a 4 per cent. manganese bronze was very little affected by heating up to 250°, and that a 15 per cent. bronze bore a temperature of 400° very well, whilst weld iron, Martin steel, copper, and delta metal did not remain uninfluenced. Subsequent analyses proved that the two bronzes contained only 3·2 and 13·5 per cent. of manganese. The present tests, made by Zizmann with cast manganese bronzes of 5·35, 7·3, 9·4 per cent., supplied by Heusler, of the Isabellenhütte near Dillenburg, confirm these results on the whole. Thirty rods of 110 mm. length and 27 mm. diameter were rolled, heated up to 100°, 200°, 300°, 400° in paraffin, tin-lead, and lead baths, and their strengths tested. A bronze of 5 or 6 per cent. would appear to answer best where tensile strength is concerned. An increased percentage of manganese affects the copper as it influences Hadfield steel, *i. e.* the strength increases first, decreases then with a further addition of manganese, and rises again with higher percentages. The turning points correspond to temperature differences of from 130° to 150°; in the poorer bronzes the changes occur at lower temperatures. Little change could be observed at 200°. H. B.

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442. *The Specific Heat of Superfused Liquids.* **L. Bruner.** (C. R. 120. pp. 912–914, 1895.)—The author experimented with organic compounds, using the method of mixtures; each experiment lasted about five minutes. The specific heat of thymol, which normally melts at 49°·5, increases with rising temperature,

most rapidly between 45° and 55°, up to 60°, and decreases slowly afterwards. Paracresol, melting-point 33°, gives a slowly ascending curve without sensible maximum; the specific heat of the superfused cresol changes little with the temperature, is about the same as that of the normal liquid, both differing in this respect from the crystals. Chloral hydrate and bromal hydrate could not be obtained in the state of superfusion, menthol only by precipitation: when the solution in acetic acid, alcohol, &c., is treated with water, the menthol (as does also thymol and cresol) is precipitated in small globules, which unite and solidify gradually. This precipitation succeeds even when the solvent has been cooled below zero; the solidification is then rapid, but can still be watched. H. B

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443. *Kinetic Energy of Heat-Motion and the corresponding Dissipation-Function.* **L. Natanson.** (Bull. Ac. Sci. Cracovie, 2. pp. 295–300, Dec. 1894; Zschr. phys. Chem. 16. pp. 289–302, 1895.)—This is an extension of a memoir of which an abstract appears on p. 6 (Abstract No. 12), employing the notation of which the author shows that approximately

$$\delta r_x/\delta t = 5\overline{\xi^2}\partial\vartheta/\partial x, \quad \&c.,$$

and

$$\partial/\partial t\,.\int\mathfrak{A}\,dV = \int\mathfrak{A}\,(lu+\ldots)\,dS + 5\int(F-p\theta)\vartheta\,dV$$
$$+\tfrac{5}{2}\int(lr_x+\ldots)p\,dS + \tfrac{5}{2}\int\{r_x\partial\vartheta/\partial x+\ldots\}\rho\,dV,$$

where

$$\mathfrak{A} \equiv \tfrac{1}{4}\rho\overline{(\xi^2+\ldots)^2}, \qquad r_x \equiv \overline{\xi(\xi^2+\ldots)}, \;\; \&c., \qquad \vartheta \equiv p/\rho \equiv \tfrac{1}{3}(\overline{\xi^2}+\ldots),$$

and l, m, n denote the direction-cosines of the *inward* normal to the element dS of the surface which encloses the volume V. $\mathfrak{A}$ may be considered the total kinetic energy of the flux of molecular energy, *i.e.* of the heat-flux, per unit volume; and its rate of variation, as the several terms of the equation show, depends on the flow of fluid into S, the viscosity of the fluid, the transformation of molar into molecular energy, the heat-gain at the surface, and lastly, since the last term can be put approximately in the form

$$\tfrac{1}{4}\int[\{\delta(\rho r_x)^2/\delta t\}/\rho\overline{\xi^2}+\ldots]\,dV,$$

on the mutual actions of the molecules on each other. This term is $\pm$ according as $r_x, \ldots$ tend to increase or diminish, and between these two cases the kinematic theory is unable to decide, though the phenomenon of conductivity requires the latter—this uncertainty being exactly analogous to that shown for viscosity in the former memoir.

If we put

$$k_x \equiv -5\rho\,\overline{\xi^2}\,r_x/(\delta r_x/\delta t), \;\; \&c.,$$

then the first equations above become

$$\rho r_x = -k_x \partial\vartheta/\partial x, \quad \&c.,$$

and an equation found in the former memoir takes the form

$$3\rho\, d\vartheta/dt - 2(\mathrm{F} - p\theta) = \partial/\partial x\,.\,(k_x\, \partial\vartheta/\partial x) + \dots$$

[wherein the author omits the coefficient 3];

further, the last term in the second equation above may be written

$$-\tfrac{5}{2}\int \{k_x(\partial\vartheta/\partial x)^2 + \dots\}\, d\mathrm{V},$$

and to complete the solution of the problem of conductivity we should have to show that

$$k_x = \dots = \text{constant.}$$

Since, then, the expression

$$-\tfrac{5}{2} k \{(\partial\vartheta/\partial x)^2 + \dots\}$$

plays the same part for conduction as does F for viscosity, we may call it the Conduction Dissipation-function. R. E. B.

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444. *Thermal Properties of Vapours.* Part 5. *Alcohol Vapour.* **A. Battelli.** (N. Cim. 4. 1. pp. 230–245, 1895.)—The same apparatus was used as in the experiments on carbon bisulphide. The critical temperature of pure alcohol is 513°·1, the critical volume 4·525 c.c., and the critical pressure 48·096 mm. The vapour-tension of alcohol at the first moment of condensation, when the temperature exceeds 50° C., is a little smaller than the maximum tension of the same vapour; the ratio between the two tensions diminishes as the temperature increases. But the ratio between the differences of tension and the corresponding diminutions of the volume of the vapour increases with the temperature. The maximum vapour-tensions of alcohol are well represented by Biot's formula between $-16°$ and $+240°$ C. The products pv, starting from saturation, increase at first with the temperature until about 140° C., after which they steadily diminish. The coefficients of expansion of alcohol vapour under constant pressure increase as the temperature falls, and the more rapidly as the vapour approaches the saturated state. When the pressure is increased, the absolute values of these coefficients increase between the same limits of temperature. The coefficients of increase of pressure for a given volume diminish as the temperature rises. As the volumes become smaller the absolute values of these coefficients become larger, and they vary more rapidly. The differences $\dfrac{p_1 v_1}{pv} - 1$ (where $p_1 v_1$ refer to the state of gas and pv to that of vapour) for each temperature increase as the vapour approaches saturation; and at the different temperatures

near saturation they increase rapidly as the temperatures rise. For alcohol, the same as for the substances previously studied, the products pv, after the substance has entered the truly gaseous state, continuously rise with the temperature. The ratio

$$\frac{p_1 v_1}{p'v'\sqrt{\mathrm{T}}}$$

of Herwig's formula (where $p'v'$ refers to the state of saturated vapour) steadily diminishes for alcohol until about 110° C., where it reaches a minimum, rising again after that. The number of molecular groups of two or more molecules which are found in alcohol-vapour at the first moment of condensation increases rapidly with the temperature when that is high; and above the critical temperature there must be formed, at sufficient pressure, triple and quadruple molecules besides the double ones. E. E. F.

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445. *Clausius' Isothermal.* **E. Riecke.** (Wied. Ann. 54. pp. 739–744, 1895.)—The method and calculation employed in a previous paper with respect to van der Waals' formula (Abstract, No. 48, p. 20) is here adapted to Clausius' equation in the form

$$p = m\mathrm{RT}(v-b)^{-1} - m^2 a\mathrm{T}^{-1}(v+c)^{-2},$$

which becomes

$$\pi = 8\theta/(3\phi'-1) - 3/\theta\phi'^2,$$

when

$$\pi \equiv p/\pi_0, \quad \theta \equiv \mathrm{T}/\theta_0, \quad \phi' \equiv (v+c)/(\phi_0+c),$$

where π_0, θ_0, ϕ_0 are the values of p, T, v in the critical state. Then the thermodynamic potential is given by

$$\mu = -\Theta + \mathrm{RT}\left(\frac{v}{v-b} - \log\frac{v-b}{m}\right) - \frac{ma(2v+c)}{\mathrm{T}(v+c)^2},$$

Θ being a function of T; and this may be written

$$\mu/\mathrm{R}\theta_0 + \mathfrak{T} = \{(\phi'-\gamma)/(\phi'-\tfrac{1}{3}) - \log(\phi'-\tfrac{1}{3})\}\theta - 9(1-\gamma/2\phi')/4\phi'\theta,$$

where $\gamma \equiv c/3(b+c)$ and $\mathfrak{T} \equiv \Theta + \mathrm{RT}\log(3\mathrm{R}\theta_0/8\pi_0)$.

Since for CO_2 Clausius finds

$$b = \cdot 000843, \quad c = \cdot 000977, \quad \text{so that } \gamma = \cdot 179,$$

and for methyl chloride Kuenen finds

$$b = \cdot 00135, \quad c = \cdot 00165, \quad \text{or } \gamma = \cdot 184,$$

Riecke takes in general for γ the mean value ·18.

Proceeding as in the former memoir, the saturated state corresponding to $\theta = \cdot 75$ is graphically found to be given by

$$\pi = \cdot 08, \quad \phi_1' = \cdot 421, \quad \phi_2' = 23{\cdot}8,$$

to which correspond $\phi_1 = \cdot 28, \quad \phi_2 = 29\cdot 5,$

where $\phi \equiv v/\phi_0 = (\phi' - \gamma)/(1-\gamma).$

The corresponding values of π, ϕ_1, ϕ_2 found before by van der Waals' formula were ·288, ·489, 5·53, while those deduced from experiments on fluorbenzene were ·115, ·415, 23·7, so that Clausius' formula better corresponds with experiment.

For mixed gases we have

$$p = (m_1 R_1 + m_2 R_2) T (v-b)^{-1} - (m_1^2 a_1 + 2 m_1 m_2 a + m_2^2 a_2) T^{-1} (v+c)^{-2},$$

and the thermodynamic potentials of the components are respectively

$$\mu_1 = -\Theta_1 + R_1 T \left(\frac{v}{v-b} - \log \frac{v-b}{m_1} \right) - \frac{(m_1 a_1 + m_2 a)(2v+c)}{T(v+c)^2},$$

$$\mu_2 = -\Theta_2 + R_2 T \left(\frac{v}{v-b} - \log \frac{v-b}{m_2} \right) - \frac{(m_1 a + m_2 a_2)(2v+c)}{T(v+c)^2};$$

and if at any temperature two different phases A and B of the mixture are in equilibrium, we must have the pressure equal throughout and

$$\mu_{1A} = \mu_{1B}, \quad \mu_{2A} = \mu_{2B},$$

whence the circumstances of the mixtures may be determined.

R. E. B.

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446. *Thermal Radiation at Low Temperatures.* **R. Pictet.** (Zschr. phys. Chem. 16. pp. 417–449, 1895.)—This paper is an account of experiments already noticed in Abstracts 239, 122, with an account of the physiological action of the low temperatures.

S. S.

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447. *Comparison of Absolute Temperatures with the Normal and Air-Thermometer Scales.* **L. Houllevigue.** (J. Phys. 4. pp. 110-118, March 1895.)—T being absolute temperature, and τ temperature on any other scale whatever,

$$\frac{T}{T_0} = \exp \int_{\tau_0}^{\tau} \frac{\partial p / \partial \tau}{J l} \, d\tau.$$

Now on a gas constant-volume thermometer,

$$p = p_0 (1 + \beta t) = \beta p_0 \tau \text{ rigorously,}$$

where β is a constant and $\tau \equiv t + \beta^{-1}$, so that

$$\partial p / \partial \tau = \beta p_0 ; \quad \text{also} \quad (C - c)/l = \partial v / \partial \tau.$$

Further, if unit mass be allowed (i) at p to contract and cool from v, τ through $dv, d\tau$, (ii) at $v - dv$ to change to $p + dp, \tau$, (iii) at τ to alter to p, v by expansion into vacuous space dv with loss of energy $J dx$, there is no loss of energy on the whole, or

$$(J C d\tau - p dv) - J c d\tau + J dx = 0,$$

where $$dv \equiv (\partial v/\partial \tau)d\tau\,;$$

but $$dx = (p'v' - pv)/\mathrm{J} - \mathrm{C}d\theta$$

if $d\theta$ is the observed cooling on expansion from p, v to p', v', and Regnault gives

$$p'v'/pv = 1 + \mathrm{A}(v' - v)/v,$$

so that $$p'v' - pv = \mathrm{A}p\,dv,$$

the cooling experiments of Lord Kelvin and Joule also giving

$$d\theta = k\tau^{-2}\,dp,$$

which the author puts in the form

$$d\theta = k\beta p_0 \tau^{-2}\,d\tau$$

[herein taking the changes of p and τ as occurring with v constant]. Hence, on reduction

$$(\partial p/\partial t)/\mathrm{J}l = a\tau^{-1} - 2b\tau^{-3},$$

where $a \equiv 1/(1-\mathrm{A})$, $b \equiv \mathrm{C}k\beta p_0/2(1-\mathrm{A})(\mathrm{C}-c)$;

and therefore, if a and b are constant as in the case of hydrogen,

$$\mathrm{T}/\mathrm{T}_0 = (\tau/\tau_0)^a \exp b(\tau^{-2} - \tau_0^{-2}),$$

or $$\mathrm{T} \propto \tau^a e^{b\tau^{-2}}.$$

Then the conditions $\mathrm{T}_{100} - \mathrm{T}_0 = t_{100} = 100$, $t_0 = 0$, give

$$\begin{aligned}\mathrm{T}_0 &= 100/\{\beta^a \tau_{100}^a \exp b(\tau_{100}^{-2} - \beta^2) - 1\}\\ &= 100/[(1+100\beta)^a \exp b\beta^2\{(1+100\beta)^{-2} - 1\} - 1]\end{aligned}$$

[τ has been introduced into the above to simplify the expressions].

For the normal hydrogen-thermometer, $\mathrm{C} = 3{\cdot}409$, $\mathrm{C}/c = 1{\cdot}3852$, $\mathrm{A} = -{\cdot}00054723$, $\beta = {\cdot}00366254$, $p_0 = 1{\cdot}3158$ atmos, $k = {\cdot}02015 \times 285^2 = 1633$, we find

$$\mathrm{T}_0 = 273{\cdot}2983\,;$$

and the maximum excess of $\mathrm{T} - \mathrm{T}_0$ over t within the range 0 to 100 is only ·001.

For Regnault's air-thermometer, $\mathrm{C} = {\cdot}23741$, $\mathrm{C}/c = 1{\cdot}406$, $\mathrm{A} = {\cdot}00110538$, $\beta = {\cdot}003665$, $p_0 = 1$, $k = 21{\cdot}281$, we have

$$\mathrm{T}_0 = 273{\cdot}3591\,;$$

and within the range 0 to 100 t exceeds $\mathrm{T} - \mathrm{T}_0$, the difference for $t = 40$ being ·0144.

R. E. B.

ELECTRICITY.

448. *Electro-bleaching.* **L. J. Matos.** (J. Frank. Inst. No. 831. pp. 177–197, March 1895.)—This paper gives a historical *résumé* of the subject with especial reference to the Hermite process on various fibres and the Kellner process for the bleaching of paper pulp. S. R.

449. *Loss of Electricity from bad Conductors near a Radiating Body.* **E. Branly.** (C. R. 120. pp. 829–831, 1895.)—The substance investigated is in the form of a disc, hanging by a wire from an electrometer, and is initially charged to 500 volts. In the presence of a red-hot cylinder a + change is rapidly dissipated, a – charge more slowly, from discs of wood, whether plain, polished or varnished, or from glass made so hot as to conduct. Thus the action here is the same as with metals. When illuminated by the more refrangible rays from an arc light or a series of sparks, a disc of wood, plain or polished, marble, card, earthenware, or hot glass is completely discharged, a – charge disappearing a little more quickly than a + charge. A disc of varnished or oiled wood loses + electricity rapidly and – electricity very slowly. Greased metal behaves similarly. J. W. C.

450. *Action of the Electrical Discharge on Lead Oxide.* **E. Warburg.** (Wied. Ann. 54. pp. 727–730, 1895.)—Lead oxide spread along a tube containing air at 2–5 mm. pressure turns brown when a discharge is sent along the tube. Manometric observations show that gas has been absorbed, and chemical tests prove that PbO_2 has been formed. In hydrogen under similar conditions the lead oxide turns black from liberation of metallic lead. In nitrogen the oxide is unaffected. The effect appears to be due to the light radiated from the glowing gas making the oxide more ready to react with the gas, and not to a chemical change in the gas which makes it more active, for the change of colour is only seen on the sides of the particles of lead oxide that are exposed to the radiation. The following experiment supports this view. A tube is divided into two compartments by an aluminium disc which serves as anode, and is pierced by a circular hole. On one side of this disc is the lead oxide, and on the other side is the kathode, and hence no conduction takes place through the compartment containing the oxide. When the current passes, the oxide changes colour at the point where radiation reaches it through the hole. J. W. C.

451. *Electrical Conduction by Hot Gases.* **E. Pringsheim.** (Sitz. Akad. Wiss. Berlin, pp. 331–334, 1895.)—A pair of circular platinum electrodes 1 cm. in diameter and 3 mm. apart are placed in a closed porcelain tube, that can be heated in a gas oven. The electrodes are connected with 1–10 dry cells and a galvanometer.

Hydrogen, carbonic acid, and air begin to conduct at a red heat, the conductivity increasing rapidly with increasing temperature or diminishing pressure. At a given temperature and pressure, the current is given by the equation

$$i = \frac{e + ae^2}{r},$$

where a is a constant depending on the nature of the electrodes and the direction of the current. After the current has passed for a time the electrodes show marked polarisation, and can of themselves send a current through the galvanometer. The maximum polarisation E.M.F. is independent of the E.M.F. of the battery, the pressure and the current. It depends on the temperature, and seems to be the same for the three gases used. The highest E.M.F. observed was ·5 volt. The polarisation disappears very slowly of itself. It is not removed by cooling and reheating, but is destroyed by a current of air. With gold electrodes the deviations from Ohm's law and the polarisation are smaller.

J. W. C.

452. *The Quadrant Electrometer as a Differential Instrument.* **R. Arnò.** (N. Cim. 4. 1. pp. 252–257, 1895.)—The paper shows how to apply a quadrant electrometer to measure capacities and self-inductions. Let A, B, C be three points, and V_A, V_B, V_C their respective potentials; if A be connected to one pair of quadrants, C to the other pair, and B to the needle, no deflection will be produced if $V_A - V_B = \pm(V_B - V_C)$. To measure capacities or self-inductions, a sinusoidal alternating E.M.F. is produced, by means of an alternator, between A and C; a variable resistance, r, free from self-induction, is placed between A and B, and between B and C the condenser or spiral, as the case may be. In the first case, let c equal the capacity of the condenser, and n the frequency of the alternator: then the needle will not be deflected if $c = 1/2\pi nr$. In the second case, let L and r' represent respectively the self-induction and resistance of the spiral; there will be no deflection if $L = \sqrt{r^2 - r'^2}/2\pi n$. The apparatus employed by the author to determine the capacity of a condenser of about one-half microfarad would detect a change in capacity of about 0·12 p. c.

A. Gs.

453. *Absolute Electrometer.* **H. Abraham** and **J. Lemoine.** (Bull. Soc. franç. Phys. No. 61.)—Two models were shown, designed for measuring very high temperatures. The standard model measures potentials up to 45,000 volts to the thousandth. The simplified model, constructed for commercial purposes, is designed to measure potentials up to 100,000 volts to within one-hundredth of their value. The standard model is constructed on Lord Kelvin's principle, with plane disc and guard-ring. It can be used either in stable or unstable equilibrium. In the simplified

model, the attracted disc is mounted on a Roberval balance. Upon the insulating rod which carries the electrified disc is stuck a glass disc to increase the insulation. When very high potentials have to be measured, the lower plate is covered with a very thin glass plate. E. E. F.

454. *Capacity Tests.* **C. Heinke.** (Wied. Ann. 54. pp. 577–603, 1895.)—If alternating currents are used in testing the capacity of condensers, the results, as is well known, vary with the frequency of alternation; and the extent of these variations is different for different dielectrics. The author finds a third source of error, which depends upon the particular method of testing and connecting with the bridge. Five condensers were compared: (I.) A standard mica condenser made by Charpentier. (II.) A paraffined paper condenser, from Cortaillod. (III.) A Swinburne condenser, intended for 1000 volts, and formed of foils, separated by eight folds of parchment paper in a liquid insulator of paraffin oil. (IV.) A condenser made after the manner of an ordinary non-inductive resistance-coil; that is to say, an insulated manganin wire, wound double upon a bobbin, but with the four ends left free. (V.) A pair of induction bobbins. The first tests relate to a comparison of I. with III., using different dispositions of the bridge and different rates of alternation. The results are given in the following table, in which $\phi = \pi z$, where z is the number of half periods in a second. A, B, C, D correspond to four orthodox bridge methods for comparing capacities. The results are supposed to be in microfarads:—

ϕ	A.	B.	C.	D.
38	1·943	1·944	3·65	1·945
92	1·543	1·544	2·76	1·558
172	1·300	1·296	2·25	1·338

The discrepancy is greatest under C, in which method the two condensers are in contiguous branches of the bridge, and are balanced against two resistances, the galvanometer being connected between the common terminal of the two condensers and the common terminal of the two resistances. There is no data with regard to the dielectric resistance of the condensers, in this test. The current was supplied from accumulators, and was alternated by a double commutator. Other things remaining the same, a considerable variation was observed in the results when the angular positions of the commutators in the battery and galvanometer branches, respectively, were altered. This led the author to a long series of tests upon the influence of the actual time of charge and the conditions of discharge. Curves are given which represent the discharges from the condensers, after successive fractional charges at a fixed rate for definite time intervals: they show the effect of the duration of charge upon the final discharge deflection. The effect of short-circuiting the condensers for different time intervals was also studied.

Special attention is called by the author to the behaviour of condensers when charged intermittently: as compared to the results when a constant difference of potentials is maintained at the terminals during the whole period of charging. It is found that an intermittent charge establishes the final condition of maximum quantity corresponding to a given voltage, in a much less time than that required by the steady application of the same voltage for the same total time of contact. This was most evident with condensers III. and IV. With I. and II. the charging was fully established too rapidly for observation. In a certain experiment, an intermittent charge of 10 contacts, each of 0·0005 second duration, produced an effect equal to about double of that of a continuous charge of 10 × 0·0005 = 0·005 sec. The author departs somewhat from the usual explanation of this phenomenon: he supposes that the repeated action of "making" and "breaking" between condenser and current source sets up an electric concussion, which is assumed to facilitate "electrification" in the same way that mechanical concussion advances the magnetisation of iron. And again, since it requires a certain time for the molecules to change their positions in the dielectric, the fractional charges, with their separating time intervals, may produce a greater total effect than would be possible from the steady strain of a continuous current. R. A.

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455. *Resonance in Alternating Current Lines.* **E. P. Houston** and **A. E. Kennelly.** (J. Am. Inst. El. Eng. 12. pp. 203–227, 1895.)—This paper is a comparison of the conditions necessary for the development of resonance in alternating current lines with those for maximum efficiency in transmission of electrical energy. The authors conclude that a loaded line is antagonistic to great efficiency in transmitting energy, since a highly resonant condenser has its pressure and current in quadrature, and the mere existence of resonance implies an elastic reflection of energy with the minimum absorption. There is an interesting figure given from which can be obtained by inspection the polar coordinates of a vector point, representing the hyperbolic sine or cosine of a given vector, whose rectangular coordinates are known. W. G. R.

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456. *New Dead-beat Voltmeters and Ammeters.* **R. Arnoux.** (L'El. 9. pp. 289–293, 1895.)—The instruments here described are a sort of combination of those made by Crompton and Weston: they consist of a moveable copper coil resting on pivots in which friction is reduced to a minimum by having the sockets made of gems. The pointer is made of aluminium. The coil moves in a strong magnetic field between the poles of a permanent tungsten steel magnet in the form of a ring. Constancy of zero is insured by means of two spirals wound in opposite directions. The author

says that a voltmeter of this description in which the moveable coil has a resistance of 75 ohms will easily indicate the sense of a current of 5 microamperes; and the sensibility is such that to measure a difference of potential of 150 volts, a resistance of at least 3000 ohms must be placed in series with the instrument. A simple method of graduating and standardising the instruments is given. The figure representing one of these instruments is apparently inaccurate, the controlling spirals being wound in the same direction. W. G. R.

457. *Pull of Magnets.* **E. Taylor Jones.** (Wied. Ann. 54. pp. 641–654, 1895.) An account of experiments made with a view of showing experimentally the $\frac{B^2}{8\pi}$ law for the normal magnetic stress in an infinitely narrow cleft perpendicular to the lines of induction in iron. An historical sketch is given describing the methods employed in the chief experiments so far made on the subject, and the results obtained, which have not shown good agreement with theory. The present experiments were made with an ovoid of soft iron cut through in the equatorial plane and with the interfaces ground truly plane and very finely polished. These experiments differ from most of the previous ones in that the induction and the stress were measured in independent experiments. The ovoid (with the two magnetising coils in which the two halves of the ovoid were axially fixed) was first laid horizontally in a stand in the east and west line, and the magnetisation curve (I and H) determined magnetically. From this curve the value of B for any value of the magnetising current could be calculated. One of the coils, with its half-core, was then suspended vertically from a tripod stand, so that the other electromagnet could be attached to it below when the current was made. By means of a guiding-ring on one of the half-cores, it was ensured that when the interfaces were in contact the half-cores together exactly made up the complete ovoid. Weights were placed in a scale-pan suspended from the lower electromagnet, and finally shot slowly poured in until separation took place. The current was then read off and the shot weighed. Thus values of the stress were measured corresponding to known values of the induction. After necessary corrections had been made the results came out within $\frac{1}{2}$ p. c. of the theoretical values. A few experiments were also made with the interfaces of the ovoid separated by very thin sheets of silver.

AUTHOR.

458. *Electrical Resistance of Sugar Solutions.* **Gin** and **Leleux.** (C. R. 120. pp. 917–920, 1895.)—Measurements by the method of Lippmann of solutions of saccharose are expressed in the form

$$y = 52160 - 3040\,x + 18980\,x^2,$$

where y = specific resistance in ohms, x = concentration in gm.-mols. per litre (temperature not stated). The temperature variation is given, but there appear to be misprints. The resistance was also found to vary with the density of current, according to the expression $y=\alpha-\beta(1+\frac{1}{2})^2$. R. A. L.

459. *Lines of Equal Disturbance in the Magnetic Potential of the Earth for* 1880. **W. von Bezold.** (Sitz. Akad. Wiss. Berlin, pp. 363–378, 1895.)—If V_n is the mean value of the magnetic potential along any given parallel of latitude, while V is the actual potential at any point on this parallel, then we may write

$$V = V_n + V_a,$$

where V_a represents the disturbance from the mean at the given point. The curve defined by

$$V_a = \text{constant}$$

is called an "isanomale," or line of equal disturbance.

The author gives the theory of the isanomales for magnetic potentials, and proves:—(1) That the westerly (or easterly) component of the earth's magnetism is given by

$$W = \frac{\partial V_a}{\partial y},$$

where ∂y is an element of a parallel of latitude. (2) That the places at which the isonamales cut the terrestrial meridian at right angles all lie on the agonic lines (*i. e.*, lines passing through the points at which the declination is zero). (3) That at places where the isonomales are parallel to the meridian, the northerly component of the earth's magnetism has the mean value corresponding to the parallel of latitude on which such a place lies. As all the isanomales are closed curves, there exist points on the earth's surface at which the disturbance is a maximum (either positive or negative). Such points the author calls the poles of the isanomales: they must all lie on the agonic lines.

A map of the isanomales of magnetic potential for 1880 is given, the necessary data having been obtained from the magnetic maps published by G. von Quintus Icilius. There are two poles of the isanomales—one in lat. 26° S. and long. 44° W., and the other in lat. 49° S. and long. 140° E. The author finds that, to a considerable degree of accuracy, the mean value of the magnetic potential for the parallel of latitude β is given by the equation

$$V_n = K \sin \beta.$$

W. W.

460. *Theory of the "Transition" Battery without Metastable Phase.* **J. H. van't Hoff, E. Cohen,** and **G. Bredig.** (Zschr. phys. Chem. 16. pp. 453–457, 1895.)—It is shown that the numbers obtained by Cohen and Bredig (Zschr. phys. Chem. 14. p. 535) for the E.M.F. of a concentration-cell consisting of mercury-mercurous sulphate electrodes in contact with a saturated solution of $Na_2SO_4,10H_2O$ (Glauber's salt) on the one hand, and with an unsaturated solution of the same salt on the other, at various temperatures, can be calculated from other data. The equation

$$\frac{dE}{dT}=\frac{E-W}{T} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

is used, where E is the E.M.F. at T. For the case in which the unsaturated solution is $\frac{1}{4}$ normal, $E=0$ when the saturated solution of Glauber's salt is also $\frac{1}{4}$ normal. This temperature may be calculated from

$$\frac{d\log C}{dT}=\frac{A}{T^2},$$

where C = the concentration of the saturated solution, and A depends on the heat of solution and may be taken as independent of the temperature. For the case considered, $E=0$ at $-16°{\cdot}2$ C. By the help of this number E may be calculated from equation (1) for other temperatures. W (the heat evolved by the process producing the current) is obtained from the heats of solution and dilution of Glauber's salt. The numbers obtained are :—

T	20°	25°	30°	33°·2
E found in $\frac{1}{1000}$ volt .	21	24·8	31·4	—
E calc. ,, ,,	22·5	26·3	30·8	34·5

At 33°·2 the hydrate $Na_2SO_4,10H_2O$ changes into the anhydrous salt and water. The value of W therefore undergoes a sudden change, the heat of solution of the anhydrous salt being 9610 cals. per equivalent greater than that of the hydrate. Allowing for this, the following numbers are obtained :—

T..................	35°	40°	45°
E found in $\frac{1}{1000}$ volt .	35·8	36·7	37·1
E calc. ,, ,,	34·8	35·3	35·8

T. E.

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461. *Resistance Variations of Bismuth.* **A. Sadovsky.** (J. Soc. Phys. Chim. Russe, 26. No. 2, 1894.)—M. Sagnac gives an account of the investigations of M. Sadovsky with regard to the changes which occur in the electrical resistance of a bismuth wire when simultaneously traversed by a current, and submitted to the action of a magnetic field. Several cases arise: the effect varies with the nature of the current employed; with the degree of magnetisation; and also with the direction of the

wire in respect to the field. The conductivity for heat of bismuth is found to fluctuate under similar circumstances.

Suppose a bismuth wire placed normally to the lines of force in a steady field of strength H: let R be its resistance as measured when a constant unidirectional current is flowing through it; and let Q be the resistance of the same wire when traversed by an alternating current. Then, when $H=0$, Q is found to be less than R; but if H is given some value greater than 6000, the reverse happens, or $R>Q$. $Q-R$ increases fairly rapidly with H: if we take $R=1$, $Q-R$ in a field of 16,000 is about 0·07; when $H=0$, $Q-R=-0·0023$. Tests made with alternating currents upon a bismuth wire, in a non-magnetic field, show that as the rate of alternation changes from 100 to 2000 $\sim$ per second, the value of $R-Q$ increases from 0·0012 to 0·0028. In a field where $H=16,000$, a wire placed normally to the lines of force, carrying a unidirectional constant current, acquires a resistance of 1·74 times its resistance corresponding to $H=0$; a result due to M. Lenard. In a still stronger field, M. Sadovsky attained for bismuth a factor of 2·2.

From tests made with $H=0$, the value of $R-Q$ for alternating currents appears to reach a minimum when the frequency, N, is about 400 $\sim$ per second; and M. Sadovsky therefore concludes that N is not the true independent variable in the phenomenon. The fact that Q (with alternating currents) differs from R (with unidirectional constant currents) is not completely explained by the mere variations of N, but rather by di/dt, the rate of change of the current, i. That is to say, Q may differ from R otherwise than by a directional change in the current; it is sufficient that the current should increase or diminish; and the inference is drawn that, generally, if the observed resistances of any body for unidirectional and alternating currents, respectively, are not equal, the resistance with increasing and decreasing currents will, likewise, be unequal. Q is thus to be regarded as a periodic function varying with the current phase.

M. Sadovsky designed an apparatus for testing the resistance of bismuth by a bridge-method, employing currents of constant sign but varying strengths. The battery-branch remains closed, and always receives the sinoidal current from a Gramme-machine, of about three periods a second. The galvanometer-branch is completed through a system of springs, which are alternately opened or closed, for any required interval, by a rotation mechanism: thus the galvanometer can be brought under the action of either a maximum, minimum, increasing, or decreasing phase of the whole period of the current from the generator. The results are, briefly, as follows:—Let R be the resistance of a bismuth wire corresponding to constant current; R′ with increasing current; R″ with maximum current; and R‴ with decreasing current. Then, when the wire is in a magnetic field, we have

$$R'>R''>R>R''',$$

and every circumstance which increases di/dt also increases $R'-R''$.

Experiments have shown, further, that the resistance is also a function of the current, of the form $\frac{1}{i}$. M. Sagnac points out that a parallel problem is the case of a resistance R, possessing self-induction, L, introduced into one of the four branches of a bridge. If the galvanometer-branch is kept closed, perfect adjustment is impossible, and only an approach can be made to a balance; but if the galvanometer-branch is opened during a very short interval only, and always at the same phase of the complete current-period, balance can be attained by adding to the branch, R, an increment r of which the sign and value depends upon the phase of i which traverses R at the moment of completing the galvanometer circuit. In this case

$$r=\frac{L}{i}\cdot\frac{di}{dt},$$

or an *apparent* resistance r varies with $1/i$ and with di/dt. So far as the tests of M. Sadovsky go, self-induction appears to have no part in the bismuth action,—a view which he justifies by the fact that the resistance variations are independent of the form given to the wire. M. Sagnac suggests that possibly inductive actions may be present to some second approximation, requiring more rigorous tests: as an example, he considers the case of an iron wire placed under similar conditions to the bismuth, and shows that the energy, represented by $Li^2/2$, may count for something in the process. R. A.

462. *Observations on Induction-Coil Sparks.* **G. T. Hanchett.** (El. World, 25. pp. 389–390, 1895.)—Details are given of microscopical and photographic observations of the spark between tinfoil terminals on a glass plate; the chief points being that the $+^{ve}$ terminal appears to be torn down, the $-^{ve}$ melted into a knob. The loss of weight was too small to determine with the balance. A glass fibre placed across the spark in such a manner as to be very sensitive to shaking, failed to show any mechanical disturbance. R. A. L.

463. *Effect of Temperature on the Discharge of a Conductor into Air.* **A. Oberbeck.** (Sitz. Akad. Wiss. Berlin, pp. 313–321, 1895.)—An earthed platinum wire that can be heated by a current is brought near a charged insulated conductor connected with an electrometer. As the wire is brought nearer the conductor, the potential of the latter falls. For a given distance between wire and conductor, the numerical value of the potential of the conductor assumes a definite stationary "discharge-potential," which is the same for a positive as for a negative charge, if the wire is at atmospheric temperature. As the temperature of the wire is moderately raised, the discharge-potential of the conductor assumes

a lower value; but if it is negatively charged, the discharging action of the wire is greater than if it is positively charged. When the wire reaches 500°, a positively-charged body is completely discharged, whilst a definite discharge-potential can be observed on a negatively-charged body up to 700°. Thus at temperatures up to 500° the wire gives off positive electricity more readily than negative, and it is only at this temperature that the well-known incapacity of a heat conductor to retain a negative charge sets in. J. W. C.

464. *Conductivity of Salts and Acids in Aqueous Solution.* **E. Franke.** (Zschr. phys. Chem. 16. pp. 463–492, 1895.)—The first part of the paper contains measurements of the conductivity in aqueous solution of 17 thallium, 2 lead, 3 cobalt, 3 nickel salts, and of 1 copper salt; also for permanganic acid and 7 permanganates. A note discusses the electrolytic separation of lead and thallium, which are both separated as peroxides at the anode. The thallium peroxide dissolves in strong nitric acid, and so it was reasonable to expect that the lead peroxide would only be separated when a nitric acid solution of lead and thallium was electrolysed. Under certain conditions the author finds that thallium is separated with the lead oxide, and that if the electrolyte be left in the voltameter both oxides redissolve, thallic salts being formed.—The second part contains measurements for certain organic acids and organic salts. S. S.

465. *Energy Transformations in Electrical Conduction.* **K. Strecker.** (Wied. Ann. 54. pp. 434–441, 1895.)—An attempt to explain the apparent difference between metallic and electrolytic conduction. Both metals and electrolytes are characterized by their ability to absorb energy from the ether; but as conduction in electrolytes is proportional to the number of ions present, we may assume that only the latter possess the ability in question. They may, at first, acquire potential energy—like that of a charged condenser; but if so, it is quickly converted into kinetic: the observed ionic movements lead to the conclusion that the direction in which the ions are moved by the ether is at right angles to the equipotential surfaces. They thus possess a certain amount of "directed" energy; but as this is continually being converted, by collisions, into the directionless energy of heat, it can only constitute a small part of the energy transformed in a moderate time. The effect, therefore, is indistinguishable from an immediate conversion of the energy of the ether into heat. But at the electrodes the movement of the ions takes place against forces (chemical attractions and repulsions) which are not the same in all directions: hence those movements may store up potential (chemical) energy in place of generating heat. The author does not, however, explain his views as to the mechanism of conduction in the metals. R. A. L.

466. *Dispersion and Dielectric Constants.* **F. Paschen.** (Wied. Ann. 54. pp. 668–674, 1895.)—The electromagnetic theory of light and the elastic-solid theory of the ether lead to equations of the same form for the case of dispersion: consequently the constants in the two sets of equations must be related to each other. The author quotes the formula obtained by Drude from Helmholtz' electromagnetic theory, and that of Ketteler from the elastic-solid theory, in the general case of dispersion. He shows that in the case of a so-called perfectly transparent body, transmitting light of all wave-lengths between λ_2 in the ultra-violet and λ_1 in the infra-red, taking no account of light more refrangible than λ_2 or less refrangible than λ_1, the equations require that the square of the refractive index calculated for infinitely long waves (n_∞) must be equal to the dielectric constant (k). Cauchy's formula for dispersion is a particular case of this, namely, that in which the spectrum of the light transmitted by the substance includes all infra-red and visible light, and possesses only one absorption-band. For such substances the relation $n^2_\infty = k$ has been verified experimentally. For water, alcohols, and crystals the relation is not satisfied. Ketteler has shown that the dispersion of water, iceland spar, flint-glass, and some other bodies which do not conform to Cauchy's equation

$$n^2 = n^2_\infty + \frac{M_1}{\lambda^2 - \lambda_1^2}$$

can be represented by Neumann's formula,

$$n^2 = n^2_\infty + \frac{M_1}{\lambda^2 - \lambda_1^2} - h\lambda^2.$$

This latter formula has only four constants, whereas the case of the substance with two absorption-bands requires five constants. Hence the formulæ tell us nothing about the absorption-bands of the substance beyond suggesting that they are determined by an expression involving only even powers of λ. In two cases, those of quartz and fluor-spar, Neumann's formula no longer suffices, so that his theory must be considered incomplete; the experimental results can only be expressed by the addition of a term involving λ^4. Even then the square of the refractive index for infinite waves comes out to be 3·568 for quartz and 6·09 for fluorspar, while the corresponding dielectric constants are 4·5 and 6·7 respectively. The discrepancy may be due to the difficulty of determining the coefficient of λ^4, or it may be owing to the existence of more than two absorption-bands. The quantity which in the electromagnetic theory represents the dielectric constant of the ether between the molecules has nearly the same value, 1·35, for quartz and fluorspar.

J. L. H.

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467. *Optical Method of observing Alternating Currents.* **J. Pionchon.** (C. R. 120. pp. 872–874, 1895.)—The method described depends on the magnetic rotation of the plane of polari-

sation of light. A solenoid traversed by the alternating current contains a long tube filled with a saturated solution of the double iodide of mercury and potassium; this tube is placed in a Laurent's half-shadow polarimeter. A beam of light from an arc-lamp is made to traverse a vessel filled with solution of potassium bichromate, and having the surface furthest from the lamp ground so as to give uniform illumination: this beam then passes through the polarimeter, and is examined intermittently after traversing the analyser. For this purpose a tuning-fork driven electrically is provided with two parallel screens, one on each prong; each screen contains a slit, and the analyser is viewed through these two slits. If the rate of the fork and the number of alternations per second of the current are nearly, but not quite, equal, the relative intensities of the light in the two halves of the field are seen to vary continuously. If the rate of the fork be known and the time between successive returns to equality of brightness be observed, the frequency of alternation of the current can be calculated. The greatest angle of rotation (θ) produced by the current can be determined thus:—Suppose the analyser turned through an angle less than θ, the field will appear of uniform brightness twice during each cycle of changes: if the angle turned through is greater than θ, the field will never be equally bright in both halves; and if the angle is equal to θ, the field will only be uniformly illuminated once in each cycle. The curve of variation of the magnetic field can be plotted by turning the analyser through different angles less than θ, and observing for each setting the interval between the times at which the two halves of the field are of equal brightness. This interval will increase as θ decreases. J. L. H.

468. *Electric Resistance at the Contact of two Metals.* **E. Branly.** (C. R. 120. pp. 869–872, 1895.)—The author takes two brass discs each about 5 cm. square, well planed and polished; between these he places thin plates of various metals of about the same size as the brass discs and similarly polished. The discs are pressed together by a weight of 6 kilos resting upon them, and the brass discs are provided with terminals for an electric current. The apparatus is placed in one arm of a Wheatstone's bridge and a balance obtained: the following results are observed. For any single metal or certain pairs of metals, such as copper and zinc, the resistance is very small, merely that of the discs, and constant; for other pairs, such as lead and aluminium, lead and iron, tin and aluminium, tin and iron, bismuth and iron, bismuth and aluminium, &c., there is an electrical contact-resistance. For instance, if a plate of aluminium be placed between two plates of bismuth, the resistance of the combination is considerable; but if the aluminium be above the bismuth plates and separated from them by a brass plate so as to avoid the aluminium-bismuth contact, the resistance is extremely small, although an extra brass plate has been added. The phenomenon is not due to polarisation, since the pile of plates causes no deflection

when connected to a galvanometer. The contact-resistance increases with time, rapidly at first, afterwards more slowly. It is diminished by putting more pressure on the plates, but it never disappears entirely. The rate of increase with time is about the same whether a current passes continuously through the plates or not. A vibration or shock produced in the neighbourhood of the plates serves to increase the resistance, whereas electric discharges decrease it. The action of the latter is somewhat uncertain, their effect being often very slight; a considerable diminution of resistance invariably occurs, however, when the plates are touched at one point by a feebly charged leyden jar. J. L. H.

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469. *Unipolar Induction.* **E. Lecher.** (Wied. Ann. 54. pp. 276–304, 1895.)—Faraday (Exp. Res. § 3090) held that when a cylindrical magnet revolves about its axis, its external lines of force "must not be considered as necessarily revolving with the magnet," but Tolver Preston has maintained that these lines revolve rigidly with the magnet. The author endeavours to decide between these two views. He first describes experiments by Hoppe, Weber, and others, and shows that either theory is competent to explain them. In the first of his own experiments he finds that the mechanical action of a cylindrical electromagnet on a current-bearing wire is the same whether the magnet be in rotation or not. Thus, if the lines of force rotate, it must be assumed that their motion does not affect their action on the current. The author next compares the current produced by "unipolar induction" when the electromagnet is rotated in free space with that obtained when the magnet is partly surrounded by thick slabs of iron. The total flux of induction through the core of the magnet is found to be increased 3·5 times by the presence of the iron, while the magnetic force at points in the air near the wire used for measuring the "unipolar induction" is diminished to $\frac{1}{5}$ of its former value. In spite of this the current induced by "unipolar induction" is found to be increased 3·5 times by the presence of the iron. If the E.M.F. is due to the motion of the lines of force through the air, these lines when they emerge from the iron must spring across the air with a speed about 17·5 times as great as that corresponding to the actual motion of the magnet. In order to find out where the E.M.F. has its origin the author employed water-dropping arrangements, but the electrification of the water by friction and other difficulties made any conclusion impossible. He next takes two cylindrical electromagnets AB,CD, each capable of revolution about a common axis, the poles B and C being of opposite signs and close together. If now the "unipolar" induction-current, obtained by making a contact at the pole D and a sliding contact at the equator of CD, be measured, it is found to be independent of the rotation of AB. This is easily understood on Faraday's theory. But practically all the lines of force from the pole D run through the air to the pole A. If CD is in rotation and AB is at rest, it is very difficult to see how

Preston's theory is to be applied unless some novel species of "lines of force" is invented in order to explain the phenomena. The author describes another experiment of a similar kind, and asks whether it is not much simpler to suppose, with Faraday, that the lines of force are unaffected by the rotation of the magnet.

[All the experiments described by the author can probably be completely explained by treating the rotating magnetic "charges" at the poles of the magnets as *magnetic convection-currents*, exactly similar to the electric convection-currents studied by Prof. Rowland. If the motion of these "charges" is "steady" (whether rectilinear, circular, or of any other type, provided that *at any fixed point in space* the magnetic force is constant), the magnetic field is exactly the same as if the charges were at rest, and in addition there will be an *electric* field which possesses a potential at all points where there is no magnetic "matter" in motion. This potential is, however, many-valued (just like the magnetic potential due to an electric current) if there is any *magnetic convection-current* passing through the curve of integration.] G. F. C. S.

470. *Determination of the Mean Temperature-Coefficient of a Magnet by means of the Sine Magnetometer.* **C. Chistoni.** (N. Cim. 4. 1. pp. 257–259, 1895.)—This is a short abstract of a paper published in the 'Atti della Societa dei Naturalisti di Modena,' s. iii. vol. xxvii. The author has examined the different methods employed for determining the temperature-coefficient and the accuracy with which the quantities, which appear in the formulæ used to calculate the results, have to be known in order to give results accurate to any given degree. No particulars of the results obtained or the methods employed are given, however, in the abstract. W. W.

471. *Experiments on Electrical Dispersion.* **P. Drude.** (Wied. Ann. 54. pp. 352–370, 1895.)—The specific inductive capacities of several substances were compared with that of air in a Hertzian resonating circuit at two different rates of oscillation. Along two parallel wires about 15 m. long, and 10 cm. apart, waves were excited by Blondlot's method, that is to say, the near ends are connected by a wire running alongside of the two wires which join the plates of the "vibrator" through the spark-gap. The far ends of the parallel wires terminated in two parallel metal plates between which the substance experimented with could be placed when desired. To these condenser-plates were connected fine wires leading to the electrodes of a Zehnder's tube, by the illumination of which the condition of resonance was indicated. A moveable wire bridged across the long parallel wires at any desired point. As this bridge was moved marked resonance occurred at definite positions along the parallel wires, each position corresponding to a definite period of vibration. If now, while the bridge is in one of these positions, a dielectric, such as alcohol, be introduced between the plates of the condenser, resonance ceases, and to

restore it the plates must be separated so as to bring the capacity down to the value it had with air. If the sp. ind. cap. of alcohol is the same under different rates of vibration-polarisation, this position of the plates will suit for all the resonating positions of the bridge previously ascertained with air. When this is not so, the alteration in the position of the condenser-plates necessary to secure resonance in any positions of the bridge will furnish a measure of the change in the sp. ind. cap. consequent on the alteration in period. The experiments appear to be most troublesome, and give results varying considerably among themselves. The author, however, considers he has obtained distinct indications of variation in sp. ind. cap. with period. In the case of ethyl alcohol or ebonite the variation is such as would indicate normal dispersion, while with water the dispersion seems to be abnormal. F. L. T.

472. *Extension of the Gaussian Potential Theory of Terrestrial Magnetism.* **L. A. Bauer.** (Proc. Am. Ass. for Adv. of Science, vol. 43. 1894.)—The author considers the question whether it is possible to derive the potential of terrestrial magnetic force at the surface of the earth for epochs where force observations are wanting, but observations of declination or inclination, or of both, are at hand. If this potential can be obtained, say, for the epochs 1600, 1700, &c., up to the present time, a secular variation term might be introduced in the Gaussian potential expression, and thus approximate magnetic charts could be drawn for any time during the interval for which the formula had been established, and various other interesting questions answered, as, for instance, What is the orbit described by the magnetic pole in the course of centuries? As the poles are the parts of the earth's surface where the potential would have a maximum or minimum value, by subjecting the potential expression to this condition the poles might be located for any particular epoch. The author gives an outline of the method by which he has investigated this point, and shows that it is theoretically possible to deduce the potential for epochs where force-observations are wanting. No attempt, however, has been made to apply these theoretical conclusions.

W. W.

473. *Velocity of Electric Waves.* **J. Trowbridge** and **W. Duane.** (Am. J. Sc. 49. pp. 297–305, April 1895.)—The following method of making this determination occurred to the authors. Increase the size and, if necessary, change the shape of an ordinary Hertzian vibrator until the period of oscillation is sufficiently long to be determined by photographing the spark, measure the length of the waves induced in a secondary circuit tuned to resonance with the vibrator; then the quotient of the wave-length by the complete period will be the velocity sought. In the primary oscillators adopted castor-oil or sheets of hard rubber were used as the dielectric between the plates of the condenser. This was charged by means of a large Ruhmkorf's coil

excited by five storage-cells. The secondaries consisted of a pair of parallel copper wires with a spark-gap at the end distant from the oscillator, and were of such length (20 to 30 m.) as to be in resonance with the primary. The intensity of the electric waves thus produced along the parallel wires of the secondary was such as to actually render them both visible and audible. That is to say, the secondary exhibited the phenomenon of stationary waves at the loops of which both a luminous discharge and a crackling sound could be detected, whereas at the nodes no such effects were present. For the further exploration of the waves in the secondary a bolometer similar to that of Paalzow and Rubens was employed. In order to determine the time of vibration a concave rotating mirror was used, and the images of the oscillating sparks thrown on a sensitive plate. The speed of the mirror was of the order 70 revolutions per second, and was determined to one part in five hundred by an electric chronograph. To avoid any error from imperfect turning of the secondary and the primary oscillator, the sparks of the secondary were photographed, since it was in the secondary that the wave-length was measured. Secondly, from the results of Bjerknes one would expect the oscillations in the secondary circuit to be much less damped than those in the primary. This expectation was realised. Photographs show ten times as many oscillations in the secondary as in the primary. This, therefore, constituted a further advantage in photographing the sparks of the secondary. The following are the significant figures obtained in the five determinations of the velocity:—2819, 2810, 2835, 2808, 2808; whence the mean value for the velocity of the electric waves along the wires is $2{\cdot}816 \times 10^{10}$ cm. per second. The authors do not think the discrepancy between this value and that usually accepted for the velocity of light can be attributed to the total error of their determination, but hope to further extend and refine their work and to publish more fully later. E. H. B.

474. *Influence of Electrical Radiation on the Resistance of Metallic Conductors.* **E. Aschkinass.** (V. phys. Ges. Berlin, 13. pp. 103–112, 1895.)—While using a bolometer made of tinfoil for the detection of electrical waves, the author discovered that the resistance was suddenly diminished about 2 per cent. at the initial exposure. This change persists, but time, mechanical jars, and heating restore the original resistance, and thus permit of convenient repetition of the phenomenon. It was found essential for the originating discharge to be oscillating in character. The effect occurs throughout the whole length of the strip of metal and not at any end connections. Similar results were obtained with wires of other metals. Very weak radiation is sufficient to produce the effect, so that it can be used as a delicate detector for such. F. T. T.

475. *Secular Change in the Earth's Magnetism.* **Alexis de Tillo.** (C. R. 120. pp. 809–812, 1895.)—From the best obtainable magnetic maps, viz., 21 maps of the isogonals for epochs between 1540 and 1885, and 7 maps of isoclinals for epochs between 1600 and 1885, the author has determined the secular variation in declination and dip, and by interpolation and extrapolation determined the values of these elements for the epochs 1550, 1600, 1650, 1700, 1750, 1800, 1850, and 1900. From the tables of secular change (of which he gives two samples, one for Europe and the other for the Atlantic), the author has drawn curves showing the path of the secular movement of the direction of the earth's field at 504 chosen points. These curves, he states, are irregular, but somewhat resemble an 8 in shape. A table is given showing for every ten degrees of latitude and longitude in Europe, and the Atlantic Ocean between the equator and lat. 30° S., the approximate values of the declination and dip for 1950. W. W.

CHEMICAL PHYSICS.

476. *Rate of Oxidation of Phosphorus, Sulphur, and Aldehyde.* **T. Ewan.** (Zschr. phys. Chem. 16. pp. 315–343, 1895.)—The velocity with which gaseous oxygen acts on these bodies, at a given temperature, depends on the pressure of the gas. After allowing for the increased rate of evaporation of the phosphorus and sulphur at lower pressures, the following results were obtained:—With phosphorus and oxygen saturated with aqueous vapour, the rate of oxidation was proportional to the pressure of the gas, up to about 550 mm. (at 20°). Above that pressure the reaction became much slower, and stopped altogether at 700 mm. With phosphorus and dried oxygen the velocity of the reaction appeared to be proportional to the square root of the pressure up to 60–70 mm., above which it also diminished, becoming zero at 200 mm. With sulphur the velocity of oxidation was proportional to the square root of the oxygen pressure up to 800 mm. (the highest pressure tried). With aldehyde it was proportional to the pressure of the aldehyde vapour and to the square root of the pressure of the oxygen up to 530 mm. (of oxygen); at higher pressures signs of a diminution in the velocity were noticed. The results can be most simply explained by supposing that the molecules of the oxygen dissociate to some small extent into atoms, and that only these atoms take part in the oxidation. The cessation of the action at higher pressures cannot as yet be explained. AUTHOR.

477. *Alloys of Definite Composition.* **H. le Chatelier.** (C. R. 120. pp. 835–837, 1895.)—Certain alloys have been obtained by purely chemical means, viz.:—by melting copper with excess of tin, and digesting with strong cold hydrochloric acid, insoluble crystals of composition $SnCu_3$ are obtained; by melting copper with excess of zinc, and digesting for a week with paste of chloride of lead, Zn_2Cu is formed. AlCu can be formed in a similar manner, but it is soluble in acid, and can only be partially separated; and a crystalline alloy of chromium and zinc can be got by Wöhler's process, and separated by chloride of lead. The compounds in question are hard, brittle, and crystalline, like metallic sulphides, and probably account for the hardness of many copper alloys. Zn_2Cu and $SnCu_3$ have been indicated by observations on the physical properties of the alloys, as, probably, definite chemical compounds; but the same has been said of $SnCu_2$ and $SnCu_4$, which the author has not been able to prepare. R. A. L.

478. *Spectra of Argon.* **M. Berthelot.** (C. R. 120. pp. 662–663, 1895.)—The author points out that the spectra of the fluorescence observed when sparking argon in presence of benzene vapour possesses certain analogies to the spectra of the aurora borealis, and suggests that the phenomena of the latter may be connected with the presence of argon in the atmosphere. S. R.

479. *Action of Hydrogen on Gelatino-bromide Photographic Plates.* **E. Cohen.** (Zschr. phys. Chem. 16. pp. 450–452, 1895.)—A metal can be precipitated from solutions of its salts by hydrogen of sufficiently high pressure; usually, however, the presence of platinum seems to be an essential condition. These experiments were undertaken to ascertain whether hydrogen at atmospheric pressure reduces silver bromide in a gelatine film, and can thus be made to act as a developer. A gelatino-bromide plate 4 cm. square was immersed in a wide-mouthed bottle full of dilute caustic-soda solution; carefully purified hydrogen was then bubbled through the solution for several hours. The plate showed no trace of reduction after 15 or 20 hours unless platinum chloride was present, but two milligrammes of platinum salt in 200 c.c. of solution sufficed to bring about a complete reduction in the same time. Platinum chloride alone, without hydrogen, gave no reduction. Exposed and unexposed plates were both reduced, but in one experiment an exposed plate yielded an image; indicating that the action proceeds at different rates in the exposed and unexposed portions. An analysis of the reduced deposit shows that it consists of silver and not platinum. Hydrogen is therefore to be included among photographic developers. J. L. H.

480. *Argon, Prout's Hypothesis and the Periodic Law.* **E. A. Hill.** (Am. J. Sc. 49. pp. 405–417, 1895.)—The author discusses the possible position of Argon in the Periodic Table of the Elements. It is difficult to find a gap for an element of atomic weight about 40, but one with an atomic weight of 20 might be placed in the VIIIth group next to Fluorine. He therefore opens the question of the validity of the reasoning for its monatomic nature from the ratio of the specific heats. Applying the principle used in astronomy for simplifying the problem of three bodies in the case of perturbations, the two atoms might be so near compared with the distance which separates molecules, that the two atoms might be physically regarded as one system. It is safer at present to consider argon diatomic, and to wait for some explanation of the supposed inconsistency between its specific heat ratio and that of other diatomic gases. Assuming that it is diatomic, it fits in with the prevalence in the natural series of the elements of a regular alternation of intervals of 3 and 1 in the values of the atomic weights. Writing the atomic weights in whole numbers, argon finds a place in the series

$$* * * * \frac{\mathrm{O}}{16} \, 3 \, \frac{\mathrm{F}}{19} \, 1 \, \frac{\mathrm{A}}{20} \, 3 \, \frac{\mathrm{Na}}{23} \, 1 \, \frac{\mathrm{Mg}}{24} \, 3 \, \frac{\mathrm{Al}}{27} * * * *.$$

A strong objection to writing the atomic weights in whole numbers is the irrationality of many of the determined ratios, notably that of H to O. The author thinks the presence of dissolved argon may be a source of error in the determination of this ratio, as argon is fairly soluble in water. S. S.

481. *Helium from Cleveite.* **P. F. Cleve.** (C. R. 120. p. 834, 1895.)—A letter from M. Clève to M. Berthelot, stating that the gas evolved from a specimen of cleveite by heating with bisulphate of potash showed the helium lines, but not those of argon.

J. W. C.

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482. *Molecular Rotation and Molecular Deviation.* **P. A. Guye.** (C. R. 120. pp. 876 877, 1895.)—There are two ways of arriving at a conception of the mean deviation due to the action of an active molecule in a solution. (1) By reference to the *molecular rotation*, a constant proposed by Landolt and others. Here it is assumed that all the liquids are reduced to the same volume containing the same number of molecules, and are examined in the polarimeter in columns of the same length. (2) By reference to the *molecular deviation* proposed by the author, who starts from the experimental fact that the polarimetric deviation depends upon the length of the column and is independent of its section. Here it is assumed that all the liquids are examined in the polarimeter in columns of such length that the polarised beam always encounters the same number of active molecules in traversing the column. Neither of these expressions gives directly a value independent of the conditions of experiment; correcting terms (which require investigation) must be applied in both cases.

The author disclaims the intention (ascribed to him by M. Aignan) of substituting a new and inexact expression in place of Biot's "specific rotatory power." The molecular deviation has a special interest in connection with certain special investigations; but he has not suggested the abandonment of Biot's specific rotation which, in spite of irregularities, is the most convenient constant for reducing polarimetric observations. M. Aignan's assertion that the formula for the molecular deviation is inexact is based upon the relation $[\alpha]=\text{const.}$ But this relation is only approximate: the quantity $[\alpha]$ depends upon (1) the temperature, (2) the solvent, and (3) the concentration. The irregularities are sometimes small, but they generally exceed the errors of observation.

D. E. J.

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483. *Estimation of Molecular Lowering of Melting-Point.* **A. Leduc.** (Bull. Soc. franç. Phys. 59. 1895.)—The curves obtained for solutions whose melting-points stand say at $-1°$ C. to $-2°$ C. differ considerably from those found, with solutions much more dilute, by Arrhenius, Pickering, and Ponsot. The principal cause of these discrepancies lies in the difficulty of measuring very small lowerings of the melting-point. Leduc immerses the freezing-vessel in a pressure-vessel containing ice and distilled water; when the solution in the freezing-vessel freezes, the pressure in the pressure-vessel is measured, and this indicates the extent to which the temperature stands below 0° C. Equilibrium is easily attained and maintained, and all corrections are eliminated, the thermometer

being dispensed with. Variations of $\frac{1}{10000}$° C. are easily dealt with, for such a variation corresponds to 1 cm. of mercury-column.

Leduc calculates the excess of pressure, δp, to which the mixture of ice and water must be exposed in terms of the osmotic pressure of the solution, and then of its concentration, using the formula of Van't Hoff for the osmotic pressure. If i be the isotonic coefficient of the substance dissolved, R the molecular constant of gases ($=8{\cdot}26\times10^7$ C.G.S.), u and u' the specific volumes of the solvent in the solid and the liquid state respectively, P the number of grammes of substance dissolved per 100 of solvent, M the molecular weight of the substance dissolved, and τ the melting-point of the solvent, the formula is

$$\delta p=\frac{iR\tau}{100\,(u'-u)}\cdot\frac{P}{M}.$$

The method enables the isotonic coefficient to be measured much more readily than by means of osmotic apparatus. A. D.

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